



Released Items

Student Name: WORKED OUT

Fall 2014
NC Final Exam
Math III



Public Schools of North Carolina
State Board of Education
Department of Public Instruction
Raleigh, North Carolina 27699-6314

Student Booklet



- 1 A board is made up of 9 squares. A certain number of pennies is placed in each square, following a geometric sequence. The first square has 1 penny, the second has 2 pennies, the third has 4 pennies, etc. When every square is filled, how many pennies will be used in total?

- A 512
 B 511
 C 256
 D 81

$n = 9$
 $a_1 = 1$
 $r = 2$

Geometric Sum

$$S_n = \frac{A_1(1-r^n)}{1-r}$$

$$S_9 = \frac{1(1-(2^9))}{1-2}$$

$$S_9 = 511$$

- 2 Let $f(x) = 14x^3 + 28x^2 - 46x$ and $g(x) = 2x + 7$. Which is the solution set to the equation $\frac{1}{12}f(x) = g(x)$?

- A $\{-3, 0, 1\}$
 B $\{-3, -1, 2\}$
 C $\{-2, 1, 3\}$
 D $\{1, 5, 11\}$

$$\frac{1}{12}(14x^3 + 28x^2 - 46x) = 2x + 7$$

$$14x^3 + 28x^2 - 46x = 24x + 84$$

$$14x^3 + 28x^2 - 70x - 84 = 0$$

$$2x^3 + 4x^2 - 10x - 12 = 0$$

(does NOT factor)

$x = -3, -1, 2$ by graphing

- 3 The equation $2x^2 - 5x = -12$ is rewritten in the form of $2(x - p)^2 + q = 0$. What is the value of q ?

A $\frac{167}{16}$

B $\frac{71}{8}$

C $\frac{25}{8}$

D $\frac{25}{16}$

$$2\left(x^2 - \frac{5}{2}x + \frac{1.5625}{1}\right) = -12 + \frac{3.125}{1}$$

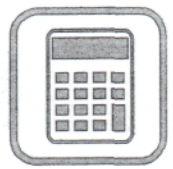
$\downarrow \qquad \uparrow$
 $\frac{5}{4} \rightarrow 1.25^2$

$$2(x - 1.25)^2 = -8.875$$

$$2(x - 1.25)^2 + 8.875 = 0$$

\downarrow

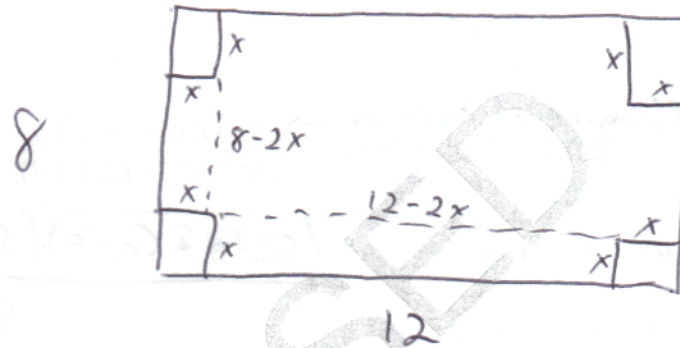
$\frac{71}{8}$



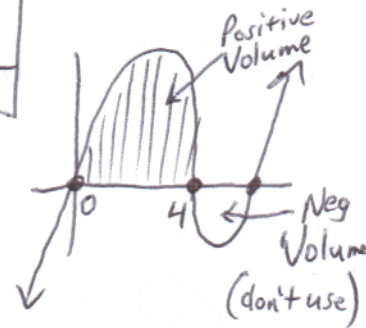
- 4 A box with an open top will be constructed from a rectangular piece of cardboard.
- The piece of cardboard is 8 inches wide and 12 inches long.
 - The box will be constructed by cutting out equal squares of side x at each corner and then folding up the sides.

What is the entire domain for the function $V(x)$ that gives the volume of the box as a function of x ?

- A $0 < x < 4$
- B $0 < x < 6$
- C $0 < x < 8$
- D $0 < x < 12$



$V = (8-2x)(12-2x)x$
 graph on Calc.



- 5 A function is shown below.

$$f(x) = \begin{cases} -x^2 + 2x & \text{for } x \leq -3 \\ 2\left(\frac{1}{3}\right)^{2x} & \text{for } -3 < x < 4 \\ \frac{2x-5}{x-7} & \text{for } x \geq 4 \end{cases}$$

What is the value of the expression $f(-3) + 2f(-1) - f(4)$?

$$-15 + 2(18) - \frac{-3}{3}$$

$$-15 + 36 + 1$$

$$22$$

- A $\frac{101}{36}$
- B $\frac{32}{9}$
- C 4
- D 22



6 Which function goes to positive ∞ most quickly as x increases?

A $y = \log(x) + 100 \rightarrow$

B $y = e^{x-9} - 3 \rightarrow$ fastest growth

C $y = x^2 + 5x + 6 \rightarrow$

D $y = 3x^5 + 4x^3 - 11x - 6 \rightarrow$

7 Which expression is equivalent to $\frac{\sin^4(\theta) - \cos^4(\theta)}{\sin^2(\theta) - \cos^2(\theta)}$, where $\sin^2(\theta) \neq \cos^2(\theta)$?
"difference of squares"

A $\sin^2(\theta) - \cos^2(\theta)$

B $\cos^2(\theta) - \sin^2(\theta)$

C 2

D 1

$$\frac{(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)}{\sin^2\theta - \cos^2\theta}$$

$$\sin^2\theta + \cos^2\theta$$

$$1$$

8 The diameter of a circle is 8 centimeters. A central angle of the circle intercepts an arc of 12 centimeters. What is the radian measure of the angle?

A $\frac{3}{2}$

B 3

C 4

D 8π

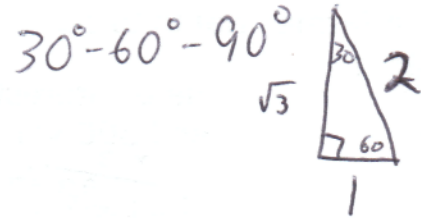
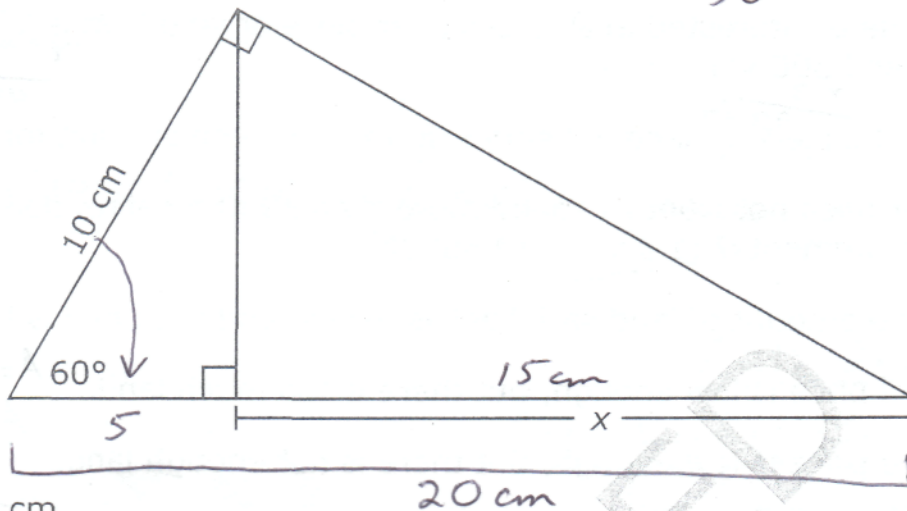
Arc Length $\rightarrow S = r\theta$ must be in radians

$$12 = 4\text{cm} \cdot \theta$$

$$3 \text{ rad} = \theta$$



9 What is the value of x in the triangle below?



A $\frac{5\sqrt{3}}{2}$ cm

B $5\sqrt{3}$ cm

C 10 cm

D 15 cm

10 To completely cover a spherical ball, a ball company uses a total area of 36 square inches of material. What is the maximum volume the ball can have?

(Note: Surface area of a sphere = $4\pi r^2$. Volume of a sphere = $\frac{4}{3}\pi r^3$.)

A 27π cubic inches

B $36\sqrt{\pi}$ cubic inches

C $\frac{36}{\sqrt{\pi}}$ cubic inches

D $\frac{27}{\pi}$ cubic inches

$$36 = 4\pi r^2$$

$$\frac{9}{\pi} = r^2$$

$$r = \frac{3}{\sqrt{\pi}}$$

$$V = \frac{4}{3}\pi \left(\frac{3}{\sqrt{\pi}}\right)^3$$

$$V = \frac{4\pi}{3} \cdot \frac{27}{\sqrt{\pi}^3}$$

$$V = \frac{4\pi \cdot 27}{3 \cdot \pi \sqrt{\pi}}$$

$$V = \frac{4 \cdot 27}{3\sqrt{\pi}} = \frac{36}{\sqrt{\pi}} \text{ in}^3$$



- 11 A farmer wants to buy between 90 and 100 acres of land.
- He is interested in a rectangular piece of land that is 1,500 yards long and 300 yards wide.
 - The piece of land is being sold as one complete unit for \$87,000.

If the farmer does not want to spend more than \$900 an acre, does the land meet all of his requirements? (1 acre \approx 43,560 ft²)

- A Yes, the amount of land satisfies his needs, and the price is low enough.
- B No, the price is low enough, but there is too much land.
- C No, the price is low enough, but there is not enough land.
- D No, the amount of land satisfies what he needs, but the price is too high.

$$\begin{array}{r} \times 3 \\ 1500 \text{ yd} \\ \hline 4500 \text{ yd} \end{array}$$

$$\begin{array}{r} \times 3 \\ 900 \text{ ft} \end{array}$$

$$A = 900 - 4500$$

$$A = 4050,000 \text{ ft}^2$$

$$\div 43,560$$

$$\frac{\$87000}{92.975} = \$935.73/\text{acre}$$

$$92.975 \text{ acres}$$

- 12 A reporter wants to know the percentage of voters in the state who support building a new highway. What is the reporter's population?
- A the number of people who live in the state *Some are too young*
- B the people who were interviewed in the state *excludes some voters*
- C all voters over 25 years old in the state *excludes 18-24 yr. olds.*
- D all eligible voters in the state

- 13 In a set of test scores that are normally distributed, a test score of 76 is 3 standard deviations below the mean. A score of 88 is 1 standard deviation above the mean. What is the mean of the data?

- A 79
- B 82
- C 84
- D 85



$$S.D. = \frac{88 - 76}{4} = 3$$

$$\text{Mean } 85$$