Introduction to Exponential and Logarithmic Functions Notes

Exponential Function: A function of the form $y = a \cdot b^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

**Exponential Functions are functions whose equations contain a variable in the exponent!!

Exponential Functions have the following characteristics:

- The functions is continuous and one-to-one
- The domain is the set of all real numbers
- The x-axis is an asymptote of the graph.
- The range is the set of all positive numbers if $a > 0$ and all negative numbers if $a < 0$.
- The graph contains the point $(0, a)$. That is the y-intercept is a.
- The graphs of $y = ab^x$ and $y = a(\frac{1}{b})^x$ are reflections across the y-axis.

Examples:	NOT Examples:
$f(x) = 2^x$	$f(x) = x^2$
$g(x) = 10^x$	$g(x) = 1^x$
$h(x) = 3^{x+1}$	$h(x) = x^x$

Logarithmic Function: The function $x = \log_b y$, where $b > 0$ and $b \neq 1$, is called a logarithmic function. This function is the inverse of the exponential function $y = b^x$ and has the following characteristics:

- The function is continuous and one-to-one.
- The domain is the set of all positive real numbers.
- The y-axis is an asymptote of the graph.
- The range is the set of all real numbers.
- The graph contains the point (1, 0). That is the x-intercept is 1.

Logarithm: In general, the inverse of $y = b^x$ is $x = b^y$. In $x = b^y$, y is called the ***logarithm*** of x. It is usually written as $y = \log_b x$ and is read "*y equals log base b of x.*"

**The inverse function of the exponential functions with base b, is called the logarithmic function with base b.
For $x > 0, b > 0, b \neq 0$,

$$b^x = y \Longrightarrow x = \log_b y$$

EXPONENTIAL FORM — LOGARITHM FORM

II. Rewriting in both forms.

Example 1) Rewrite logarithmic each equation in its equivalent exponential form.

a. $\log_5 x = 2$ $\quad 5^2 = x$

b. $\log_3 7 = y$ $\quad 3^y = 7$

c. $2 = \log_b 25$ $\quad b^2 = 25$

d. $3 = \log_b 64$ $\quad b^3 = 64$

e. $3 = \log_7 x$ $\quad 7^3 = x$

f. $\log_4 26 = y$ $\quad 4^y = 26$

Example 2) Rewrite each exponential equation in its equivalent logarithmic form.

a. $12^2 = x$ $\log_{12} x = 2$

b. $2^5 = x$ $\log_2 x = 5$

c. $8^3 = c$ $\log_8 c = 3$

d. $b^3 = 8$ $\log_b 8 = 3$

e. $b^3 = 27$ $\log_b 27 = 3$

f. $4^y = 9$ $\log_4 9 = y$

IV. **Basic and Inverse Log Properties-** Because logs are exponents, they have properties that can be verified using the properties of exponents.

Basic Properties:

1. $\log_b b = 1$ because $b^1 = b$
2. $\log_b 1 = 0$ because $b^0 = 1$

Inverse Properties: (Cancel with the same base!)

1. $\log_b b^x = x$
2. $b^{\log_b x} = x$

Example 3) Evaluate using the log properties.

a. $\log_7 7$ $= 1$

b. $\log_5 1$ $= 0$

c. $\log_4 4^5$ $= 5$

d. $\log_7 7^8$ $= 8$

e. $\log_9 9$ $= 1$

f. $\log_8 1$ $= 0$

g. $6^{\log_6 9}$ $= 9$

h. $3^{\log_3 17}$ $= 17$

Common Logarithm: Base 10 Logarithm, usually written **without the subscript 10.**
$\log_{10} x = \log x, x > 0$. Most calculators have a $\boxed{\text{LOG}}$ key for evaluating common logarithms. The calculator is programmed in base 10.

Example 4) Find the value of each log. Round to the nearest ten-thousandths.

a. $\log 81{,}000$ $= 4.9085$

b. $\log 6$ $= .7782$

c. $\log 0.35$ $= -.4559$

d. $\log 0.0027$ $= -2.5686$

Evaluating Logs using the Change of Base Formula

For all positive numbers, a, b, and n, where $a \neq 1$ *and* $b \neq 1$,

$$\log_a n = \frac{\log_b n}{\log_b a}$$

Example: $\log_5 12 = \frac{\log_{10} 12}{\log_{10} 5}$

This formula allows us to evaluate a logarithmic expression of any base by translating the expression into one that involves common logarithms.

Example 5) Evaluate each logarithm

a. $\log_3 18$ 2.6309

b. $\log_4 25$ 2.3219

c. $\log_2 16$ 4

d. $\log_{25} 5$ .5

e. $\log_2 1024$ 10

f. $\log_5 125$ 3

II. **Solving for variables with exponentials and logs.**

****MAY HAVE TO REWRITE AND APPLY PROPERTIES OR CHANGE OF BASE FORMULA!!!

Example 6) Solve for the variable:

a. $\log_3 243 = y$

$y = 5$

b. $\log_9 x = -3$

$9^{-3} = x$

$\frac{1}{9^3} = x$

$\frac{1}{729} = x$

c. $\log_8 n = \frac{4}{3}$

$8^{4/3} = n$

$16 = n$

Example 7) Evaluate:

a. $\log_8 8^4 = x$

$x = 4$

b. $\log_9 9^2 = y$

$y = 2$

Example 8) Solve each log equation. Be sure to check your answers!

a. $\log_3(3x - 6) = \log_3(2x + 1)$

$3x - 6 = 2x + 1$

$x = 7$

b. $\log_6(3x - 1) = \log_6(2x + 4)$

$3x - 1 = 2x + 4$

$x = 5$

c. $\log_8(x^2 - 14) = \log_8(5x)$

$x^2 - 14 = 5x$

$x^2 - 5x - 14 = 0$

$(x - 7)(x + 2) = 0$

$x = 7$ $x = -2$

You can't take a log of negative number...error!

d. $\log_4 x^2 = \log_4(4x - 3)$

$x^2 = 4x - 3$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3$ $x = 1$

e. $\log_5(x - 7) = 2$

$5^2 = x - 7$

$25 = x - 7$

$32 = x$

f. $\log_2(4x + 1) = 5$

$2^5 = 4x + 1$

$32 - 1 = 4x$

$\frac{31}{4} = \frac{4x}{4}$

$7.75 = x$

Math 3 Name ______________

Notes: Intro to Logarithms

Unit 4 Day 2

Solving $t = 3^{20}$ on the calculator is much easier than solving $3^t = 20$. Why?

When the unknown is in the exponent and it is not easy to make the bases equal, we need to be able to re-write the equation into a form solved for the exponent.

If $3^t = 20$, put in your own words what this equation should mean:

t = is the power of base 3 that is equal to 20 *NOW INVESTIGATE

=2.727

Definition of Logarithm with Base *b*

Let *b* and *y* be *positive* numbers with $b \neq 1$.

The **logarithm of *y* with base *b*** is denoted by: $\log_b(y)$ argument

$\log_b(y) = x$ **if and only if** $b^x = y$

(logarithmic form) **(exponential form)**

$x = \log_b y$ is read X equals log base b of y

The numbers that appear have special names:

x is the logarithm (the exponent)

b is the base

y is the argument

Review the restrictions on b and x in the definition above. Explain why each is necessary:

1) $b \neq 1$: 1 to any power is 1, so x could be anything

$-1^1 = x$ 2) $b > 0$: logs are only defined for positive real bases $y = \log_{-4} 64$

$\log_5(-25) = x$ 3) $y > 0$: impossible to take log of a negative number

no solution for negative argument

$(-4)^3 = 64$

no solution

Rewrite the following equations in the missing form.

	Logarithmic Form	*Exponential Form*
1.	$\log_2 8 = 3$	$2^3 = 8$
2.	$\log_4 1 = 0$	$4^0 = 1$
3.	$\log_{12} 12 = 1$	$12^1 = 12$
4.	$\log_{\frac{1}{4}} 4 = -1$	$\left(\frac{1}{4}\right)^{-1} = 4$
5.	$\log_{\frac{1}{2}} 32 = -5$	$\left(\frac{1}{2}\right)^{-5} = 32$
6.	$\log_3 81 = 4$	$3^4 = 81$

Evaluate each logarithm *without* a calculator.

* Ask yourself- what power of *b* gives you *y*?

7. $\log_4 64$ or $4^x=64$ $4^x=4^3$ $x=3$	8. $\log_2 \frac{1}{16} = -4$	9. $\log_{\frac{1}{3}} 27$ $= -3$
10. $\log_7 1 = 0$	11. $\log_5 5 = 1$	12. $\log_{36} 6 = \frac{1}{2}$
13. $\log_2 \frac{1}{128} = -7$	14. $\log_{32} 2 = \frac{1}{5}$	15. $\log_8 8^4 = 4$

2
4
16
32
64
128

SPECIAL LOGARTIHMS

Common Logarithm	Natural Logarithm
➢ Logarithm with base 10	➢ Logarithm with base *e*
➢ Denoted by: $\log_{10} x$	➢ Denoted by: $\log_e x$
➢ Simplified Notation: $\log x$	➢ Simplified Notation: $\ln x$

Special Note: your calculator has keys for evaluating the common and natural logarithm.

Evaluate each logarithm *without* a calculator.

16. $\log 1000$ 3	17. $\ln e^7$ 7	18. $\log \frac{1}{10000}$ -4

Evaluate each logarithm *with a calculator.*

19. ln 14 2.64, so $e^{2.64}=14$	20. log 580 2.76	21. $\frac{\ln 15}{2-\ln 10} = -8.95$

We now know that a logarithm is perhaps best understood as being closely related to an exponential equation. In fact, whenever we get stuck in the problems that follow we will return to this one simple insight.

- When working with logarithms, if ever you get stuck, try rewriting the problem in exponential form.
- Conversely, when working with exponential expressions, if ever you get stuck, try rewriting the problem in logarithmic form.

Think:
Type of problem? *Variable is in the exponent.*
Technique: *Try to make bases* equal *Then set exponents* equal *and solve for* variable.

22) $\log_5 \frac{1}{25} = y$

$5^y = \frac{1}{25}$

$5^y = 5^{-2}$

$y = -2$

*23) $\log_7 7^2 = x$

$7^x = 7^2$

$x = 2$

*24) $\log_3 1 = x$

$3^x = 1$

$x = 0$

What if the variable is not the exponent??

Find the base, x, of the logarithm without calculator:

Think:
Type of problem? *Variable is the base.*
Technique: *Raise both sides to* reciprocal *power and solve for the variable.*

Question:
Will the value of x be + and - ?

25) $\log_x 2 = \frac{1}{3}$

$(x^{1/3})^3 = (2)^3$

$x = 8$

26) $\log_x \frac{4}{9} = \frac{2}{3}$

$(x^{2/3})^{3/2} = (4/9)^{3/2}$

$x = \frac{8}{27}$

Find the argument, x, of the logarithm without calculator:

30) $\log_{25} x = \frac{3}{2}$

Question: Will the value of x be + and - ?

$25^{3/2} = x$

Solve for x:

*31) $\log_8(7x - 9) = \log_8(2x + 1)$

$7x - 9 = 2x + 1$

$5x = 10$

$x = 2$

Evaluate without a calculator:

*32) $5^{\log_5 25}$

25

33) $5^{\log_5 7}$

7

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CW/HW

Honors CC3
Notes: Properties of Logarithms

Name:________________
Date :________________

Recall: $x = b^y$ is equivalent to $\log_b(x) = y$

I. Rewrite each of the following in logarithmic form.

1. $3^4 = 81$ $\log_3 81 = 4$
2. $2^6 = 64$ $\log_2 64 = 6$
3. $5^3 = 125$ $\log_5 125 = 3$
4. $8^0 = 1$ $\log_8 1 = 0$
5. $4^{-2} = \frac{1}{16}$ $\log_4 \frac{1}{16} = -2$
6. $3^{-1} = \frac{1}{3}$ $\log_3 \frac{1}{3} = -1$
7. $7^{-2} = \frac{1}{49}$ $\log_7 \frac{1}{49} = -2$
8. $3^{\frac{1}{2}} = \sqrt{3}$ $\log_3 \sqrt{3} = \frac{1}{2}$
9. $9^{\frac{3}{2}} = 27$ $\log_9 27 = 3/2$

II. Rewrite each of the following in exponential form.

1. $\log_2 32 = 5$ $2^5 = 32$
2. $\log_8 64 = 2$ $8^2 = 64$
3. $\log_{11} 121 = 2$ $11^2 = 121$
4. $\log_5 1 = 0$ $5^0 = 1$
5. $\log_3 243 = 5$ $3^5 = 243$
6. $\log_{\frac{1}{2}} 16 = -4$ $(\frac{1}{2})^{-4} = 16$
7. $\log_8 4 = \frac{2}{3}$ $8^{2/3} = 4$
8. $\log_{10} \frac{1}{10} = -1$ $10^{-1} = \frac{1}{10}$
9. $\log_{27} 3 = \frac{1}{3}$ $27^{1/3} = 3$

III. Solve for x in each of the following equations.

1. $\log_x 64 = 3$ 4
   $\sqrt[3]{x^3}\ \sqrt[3]{64}$
   $x = 4$
2. $\log_x 49 = 2$ 7
   $x^2 = 49$
3. $\log_6 x = 2$ 36
   $6^2 = x$
4. $\log_9 x = -1$ 1/9
   $9^{-1} = x$
   $\frac{1}{9}$
5. $\log_{\frac{1}{2}} 16 = x$ −4
   $x = \frac{\log 16}{\log 1/2}$
6. $\log_3 27 = x$ 3
   log BASE
7. $\log_x 8 = 1$ 8
   $x^1 = 8$
8. $\log_5 x = -2$ 1/25
   $5^{-2} = x$
   $\frac{1}{5^2} = \frac{1}{25}$
9. $\log_5 \sqrt{5} = x$ 1/2
   $5^x = \sqrt[2]{5}$
   $5^x = 5^{1/2}$
10. $\log_2 x = -4$ 1/16
    $2^{-4} = x$
    $\frac{1}{2^4}$
    $\frac{1}{1}$
11. $\log_x \sqrt[3]{7} = \frac{1}{3}$ 7
    $x^{1/3} = \sqrt[3]{7}$
    $x^{1/3} = 7^{1/3}$
12. $\log_{\frac{1}{2}} x = 3$ 1/8
    $(\frac{1}{2})^3 = x$

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Name________________________________

Properties of Logarithms

Date________________ Period____

Expand each logarithm.

1) $\log (6 \cdot 11)$

$\log 6 + \log 11$

2) $\log (5 \cdot 3)$

$\log 5 + \log 3$

3) $\log \left(\frac{6}{11}\right)^5$

$5\log 6 - 5\log 11$

4) $\log (3 \cdot 2^3)$

$\log 3 + 3\log 2$

5) $\log \frac{2^4}{5}$

$4\log 2 - \log 5$

6) $\log \left(\frac{6}{5}\right)^6$

$6\log 6 - 6\log 5$

7) $\log \frac{x}{y^6}$

$\log x - 6\log y$

8) $\log (a \cdot b)^2$

$2\log a + 2\log b$

9) $\log \frac{u^4}{v}$

$4\log u - \log v$

10) $\log \frac{x}{y^5}$

$\log x - 5\log y$

11) $\log \sqrt[3]{x \cdot y \cdot z}$

$\frac{\log x}{3} + \frac{\log y}{3} + \frac{\log z}{3}$

12) $\log (x \cdot y \cdot z^2)$

$\log x + \log y + 2\log z$

7A

Condense each expression to a single logarithm.

13) $\log 3 - \log 8$

$\log \frac{3}{8}$

14) $\frac{\log 6}{3}$

$\log \sqrt[3]{6}$

15) $4\log 3 - 4\log 8$

$\log \frac{3^4}{8^4}$

16) $\log 2 + \log 11 + \log 7$

$\log 154$

17) $\log 7 - 2\log 12$

$\log \frac{7}{12^2}$

18) $\frac{2\log 7}{3}$

$\log \sqrt[3]{7^2}$

19) $6\log_3 u + 6\log_3 v$

$\log_3 (v^6u^6)$

20) $\ln x - 4\ln y$

$\ln \frac{x}{y^4}$

21) $\log_4 u - 6\log_4 v$

$\log_4 \frac{u}{v^6}$

22) $\log_3 u - 5\log_3 v$

$\log_3 \frac{u}{v^5}$

23) $20\log_6 u + 5\log_6 v$

$\log_6 (v^5u^{20})$

24) $4\log_3 u - 20\log_3 v$

$\log_3 \frac{u^4}{v^{20}}$

Critical thinking questions:

25) $2(\log 2x - \log y) - (\log 3 + 2\log 5)$

$\log \frac{4x^2}{75y^2}$

26) $\log x \cdot \log 2$

Can't be simplified.

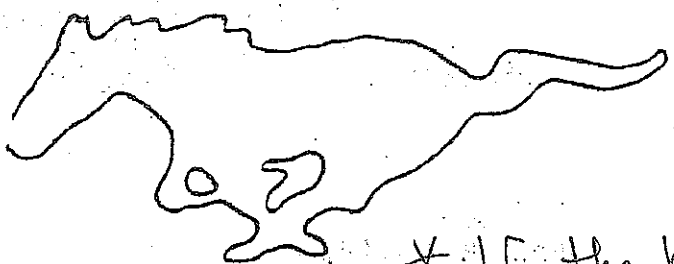
TB

Solving Exponential Equations Notes

How do I solve exponential equations when the bases are the same?

How do I solve exponential equations when the bases are different?

$2^{-4} = 1/16$	$3^{-4} = 1/81$	$4^{-4} = 1/256$	$5^{-4} = 1/625$	$6^{-4} = 1/1296$	$7^{-4} = 1/2401$
$2^{-3} = 1/8$	$3^{-3} = 1/27$	$4^{-3} = 1/64$	$5^{-3} = 1/125$	$6^{-3} = 1/216$	$7^{-3} = 1/343$
$2^{-2} = 1/4$	$3^{-2} = 1/9$	$4^{-2} = 1/16$	$5^{-2} = 1/25$	$6^{-2} = 1/36$	$7^{-2} = 1/49$
$2^{-1} = 1/2$	$3^{-1} = 1/3$	$4^{-1} = 1/4$	$5^{-1} = 1/5$	$6^{-1} = 1/6$	$7^{-1} = 1/7$
$2^{0} = 1$	$3^{0} = 1$	$4^{0} = 1$	$5^{0} = 1$	$6^{0} = 1$	$7^{0} = 1$
$2^{1} = 2$	$3^{1} = 3$	$4^{1} = 4$	$5^{1} = 5$	$6^{1} = 6$	$7^{1} = 7$
$2^{2} = 4$	$3^{2} = 9$	$4^{2} = 16$	$5^{2} = 25$	$6^{2} = 36$	$7^{2} = 49$
$2^{3} = 8$	$3^{3} = 27$	$4^{3} = 64$	$5^{3} = 125$	$6^{3} = 216$	$7^{3} = 343$
$2^{4} = 16$	$3^{4} = 81$	$4^{4} = 256$	$5^{4} = 625$	$6^{4} = 1296$	$7^{4} = 2401$
$2^{5} = 32$	$3^{5} = 243$	$4^{5} = 1024$	$5^{5} = 3125$		
$2^{6} = 64$	$3^{6} = 729$	$4^{6} = 4096$			
$2^{7} = 128$	$3^{7} = 2187$				



Solving Logarithmic and Exponential Equations

Solve for x.

* If the bases are equal, then the powers are equal.
* Reduce to same base if possible.

① $5^{x} = 5^{2x-5} \Rightarrow x = 2x-5 \Rightarrow 5 = x$

② $25^{3x+3} = 125 \Rightarrow (5^2)^{3x+3} = 5^3$

$5^{6x+6} = 5^3 \Rightarrow 6x+6 = 3 \Rightarrow 6x = -3 \Rightarrow x = -\frac{1}{2}$

③ $3^{2-x} = \frac{1}{3} \Rightarrow 3^{2-x} = 3^{-1}$

$2 - x = -1$

$3 = x$

④ $16^{2b+3} = \frac{1}{64} \Rightarrow (2^4)^{2b+3} = 2^{-6}$

2

$2^{8b+12} = 2^{-6}$

$8b + 12 = -6 \rightarrow 8b = -18$

$b = \frac{-18}{8} = \frac{-9}{4} = -2\frac{1}{4}$

⑤ $6^{x} = 45$ * If you cannot reduce to the same base, then use log function or convert to logs.

⑥ $5^{x+3} = 17$

* convert to log form

⑥ $\log_5 17 = x + 3$

$(\log_5 17) - 3 = x$

$-1.24 = x$

⑤ $6^{x} = 45$

$\log 6^{x} = \log 45$

$x \cdot \log 6 = \log 45$

$x = \frac{\log 45}{\log 6} = 2.12$

or

$\log_6 45 = x$

$\frac{\log 45}{\log 6} = x$

$x = 2.12$

⑦ $5^{x-1} = 3^{x}$

Take log of both sides.

$\log 5^{(x-1)} = \log 3^{(x)}$

$(x-1) \log 5 = x \log 3$

$x \log 5 - 1 \log 5 = x \log 3$

$x \log 5 - x \log 3 = 1 \log 5$

$x(\log 5 - \log 3) = 1 \log 5 \Rightarrow$

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Exponential Equations Not Requiring Logarithms

Date________________ Period____

Solve each equation.

1) $4^{2x+3} = 1$

$\left\{-\frac{3}{2}\right\}$

2) $5^{3-2x} = 5^{-x}$

$\{3\}$

3) $3^{1-2x} = 243$

$\{-2\}$

4) $3^{2a} = 3^{-a}$

$\{0\}$

5) $4^{3x-2} = 1$

$\left\{\frac{2}{3}\right\}$

6) $4^{2p} = 4^{-2p-1}$

$\left\{-\frac{1}{4}\right\}$

7) $6^{-2a} = 6^{2-3a}$

$\{2\}$

8) $2^{2x+2} = 2^{3x}$

$\{2\}$

9) $6^{3m} \cdot 6^{-m} = 6^{-2m}$

$\{0\}$

10) $\frac{2^x}{2^x} = 2^{-2x}$

$\{0\}$

11) $10^{-3x} \cdot 10^{x} = \frac{1}{10}$

$\left\{\frac{1}{2}\right\}$

12) $3^{-2x+1} \cdot 3^{-2x-3} = 3^{-x}$

$\left\{-\frac{2}{3}\right\}$

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Solving Exponential Equations with Logarithms

Date________________ Period____

Solve each equation. Round your answers to the nearest ten-thousandth.

1) $3^{b} = 17$

2.5789

2) $12^{r} = 13$

1.0322

3) $9^{n} = 49$

1.7712

4) $16^{v} = 67$

1.5165

5) $3^{a} = 69$

3.854

6) $6^{r} = 51$

2.1944

7) $6^{n} = 99$

2.5646

8) $20^{r} = 56$

1.3437

9) $5 \cdot 18^{6x} = 26$

0.0951

10) $e^{x-1} - 5 = 5$

3.3026

11) $9^{n+10} + 3 = 81$

-8.0172

12) $11^{n-8} - 5 = 54$

9.7005

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Solving equations with logarithms on both sides of the equation (no constants).

Property of Equality:

Examples: Cancel the log on both sides

a. $\cancel{\log_6} x = \cancel{\log_6} 5$

$x = 5$

b. $\cancel{\log_3}(x-1) = \cancel{\log_3}(2x+5)$

$x - 1 = 2x + 5$

$\cancel{-6 = x}$ no solution

c. $\cancel{\log}(p^2 - 2) = \cancel{\log} p$

$p^2 - 2 = p$

$p^2 - p - 2 = 0$

$(p+1)(p-2) = 0$

$\cancel{p = -1}$ $p = 2$

* You can't take the log of negative number.

d. $\log_3 x + \log_3 (x-6) = \log_3 16$

$\cancel{\log} x(x-6) = \cancel{\log_3} 16$

$x^2 - 6x = 16$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8$ $\cancel{x = -2}$

e. $\log(x+3) - \log(2x-4) = \log 3$

$\cancel{\log} \frac{x+3}{2x-4} = \cancel{\log} \frac{3}{1}$

$3(2x-4) = x+3$

$6x - 12 = x + 3$

$5x = 15$

$x = 3$

Steps:

1. Simplify all of the logarithms using the properties of logarithms.
2. If all of the bases are the same, then use the equality property of logarithms.
3. Solve and check for extraneous solutions $b>0$ and $x>0$ for $\log_b x = y$

$b \neq 1$ typo

Practice

1. $\log_2(8-6x) = \log_2 32$

$8 - 6x = 32$

$-6x = 24$

$x = -4$

2. $\log_{10}(x+9) + \log_{10} x = \log_{10} 10$ $x = 1$

3. $\log_4(x+3) - \log_4(x-5) = \log_4 16$

$\cancel{\log_4} \frac{x+3}{x-5} = \cancel{\log_4} 16$

$16(x-5) = x+3$

$16x - 80 = x + 3$

$\frac{15x}{15} = \frac{83}{15}$

$x = \frac{83}{15} = 5.5\overline{3}$

4. $\log_2(x+3) + \log_2(x-3) = \log_2 16$ $x = 5$

5. $\log_7(x+2) + \log_7(x+1) = \log_7 6$

$\cancel{\log_7}(x+2)(x+1) = \cancel{\log_7} 6$

$x^2 + 3x + 2 = 6$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$\cancel{x = -4}$ $x = 1$

6. $\log_6(x+3) + \log_6(x+2) = \log_6 20$ $x = 2$

7. $2\log_3 x^2 = \log_3 4 + \log_3(x+8)$

$\log_3 x^2 = \log_3 4(x+8)$

$x^2 = 4x + 32$

$x^2 - 4x - 32 = 0$

$(x-8)(x+4) = 0$

$x = 8$ $\cancel{x = -4}$

8. $2\log_b(t) - \log_b 2 = \log_b(2t+6)$ $t = 6$

$2 \cdot \log_b \frac{t^2}{2} = \log_b(2t+6) \cdot 2$

$t^2 = 4t + 12$

$t^2 - 4t - 12 = 0$

$(t-6)(t+2) = 0$

$t = 6$ $\cancel{t = -2}$

12

Solving equations with logarithms on one side of the equation (constants are visible).

Examples: Convert to exponentials

a. $\log_3(x-7)=2$

$3^2 = x-7$

$9 = x-7$

$16 = x$

b. $\log_{12}(2x-1)+\log_{12}(x-3)=1$

$\log_{12}(2x-1)(x-3) = 1$

$12^1 = (2x-1)(x-3)$

$12 = 2x^2-6x-x+3$

$0 = 2x^2-7x-9$

$0 = (2x-9)(x+1)$

$2x = 9$ $\quad \cancel{x=-1}$

$x = \frac{9}{2}$

$x = 4\frac{1}{2}$

Steps:

1. Use all properties of logarithms to simplify to one logarithm.
2. Convert to an exponential equation.
3. Solve and check for extraneous solutions $b>0$ and $x>0$ for $\log_b x = y$

?

Practice:

1. $\log_2(x^2-9)=4$

$2^4 = x^2-9$

$16 = x^2-9$

$25 = x^2$

$5 = x$

$-5 = x$

2. $\log_2(y+2)-\log_2(y-2)=1$ $\quad y=6$

$\log_2 \frac{y+2}{y-2} = 1$

$2^1 = \frac{y+2}{y-2}$

3. $\log_5(5x+5)-\log_5(x^2-1)=0$

$\log_5 \frac{5x+5}{x^2-1} = 0$

$\log_5 \frac{5\cancel{(x+1)}}{\cancel{(x+1)}(x-1)} = 0$

$5^0 = \frac{5}{x-1}$

$1 = \frac{5}{x-1}$

$x-1 = 5$ $\quad x=6$

4. $2\log_3 x-\log_3(x-2)=2$ $\quad x=3 \quad x=6$

$\log_3 \frac{x^2}{x-2} = 2$

$\frac{3^2}{1} = \frac{x^2}{x-2}$

$x^2 = 9x-18$

$x^2-9x+18=0$

5. $\log_6(x^2+2)+\log_6 2=2$

$\log_6 (x^2+2)(2) = 2$

$6^2 = 2x^2+4$

$36 = 2x^2+4$

$32 = 2x^2$

$16 = x^2$

$\pm 4 = x$

6. $\log_8(x+6)+\log_8(x-6)=2$ $\quad x=10$

15

Solving Log Equations Using Properties

1. $ln_e(2x + 4) = 3$

$e^3 = 2x+4$

$\frac{e^3 - 4}{2} = \frac{2x}{2}$

$8.0428 = x$

2. $log_5 2 + log_5 x = 3$

$\log_5 2x = 3$

$\frac{5^3}{2} = \frac{2x}{2}$

$62.5 = x$

3. $log_8 4x^4 - log_8 2x^2 = 1$

$\log_8 \frac{4x^4}{2x^2} = 1$

$\log_8 2x^2 = 1$

$\frac{8^1}{2} = \frac{2x^2}{2}$

$4 = x^2$

$2 = x$

$-2 = x$

4. $log_4(10x - 8) = log_4(x + 4)$

$10x - 8 = x + 4$

$\frac{9x}{9} = \frac{12}{9}$

$x = \frac{4}{3}$

5. $log_3(x + 10) - log_3 x = 4$

$\log_3 \frac{x+10}{x} = 4$

$3^4 = \frac{x+10}{x}$

$\frac{81}{1} = \frac{x+10}{x}$

$81x = x+10$

$\frac{80x}{80} = \frac{10}{80}$

$x = \frac{1}{8}$

6. $log_2 x + log_2(x + 6) = 4$

$\log_2 x(x+6) = 4$

$2^4 = x(x+6)$

$16 = x^2 + 6x$

$0 = x^2 + 6x - 16$

$0 = (x+8)(x-2)$

$\cancel{x = -8}$

$x = 2$

7. $ln x + ln x^2 = 21$

$ln_e\, x(x^2) = 21$

$e^{21} = x^3$

$\sqrt[3]{e^{21}} = \sqrt[3]{x^3}$

$x = e^7$

$x = 1096.6332$

8. $log_7 x^2 = log_7(x + 20)$

$x^2 = x + 20$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x = 5$ $\quad x = -4$

9. $log_4(x + 4) + log_4(x + 64) = 4$

$\log_4 (x+4)(x+64) = 4$

$4^4 = (x+4)(x+64)$

$256 = x^2 + 68x + 256$

$0 = x^2 + 68x$

$x(x+68) = 0$

$x = 0$ $\quad x = -68$

10. $ln_e(3x - 8) = 2$

$e^2 = 3x - 8$

$\frac{e^2 + 8}{3} = \frac{3x}{3}$

$5.1297 = x$

14

Solving Exponential & Logarithmic Equations 2

Name: ______ Date: ______ Pd: ____

Use the rules of exponents or logarithms to find the value of x in each equation.

1. $(5^{x+1})^5 = 5^{25x}$

$5^{5x+5} = 5^{25x}$
$5x+5 = 25x$
$5 = 20x$
$\frac{5}{20} = \frac{20x}{20}$
$\frac{1}{4} = x$

2. $(9^{2x})(9^{16}) = 9^{48}$

$9^{2x+16} = 9^{48}$
$2x+16 = 48$
$2x = 32$
$x = 16$

3. $\frac{4^{50}}{4^{40}} = 4^{x-5}$

$4^{10} = 4^{x-5}$
$10 = x-5$
$15 = x$

4. $(64^2)(16^x) = 4^{12}$

$4^{3 \cdot 2} \cdot 4^{2x}$
$4^6 \cdot 4^{2x}$
$4^{6+2x} = 4^{12}$
$6+2x = 12$
$2x = 6$
$x = 3$

5. $\left(16^{\frac{1}{2}}\right)(2^3) = x$

$2^{4 \cdot \frac{1}{2}} \cdot 2^3 = x$
$2^2 \cdot 2^3 = x$
$2^5 = x$
$32 = x$

6. $(25^3)(5^6) = 125^{3x-2}$

$5^{2 \cdot 3} 5^6 = 5^{3(3x-2)}$
$5^{12} = 5^{9x-6}$
$12 = 9x-6$
$6 = 9x$
$x = \frac{6}{9} = \frac{2}{3}$

7. $10^{2x-1} = 100$

$10^{2x-1} = 10^2$
$2x-1 = 2$
$2x = 3$
$x = 3/2$

8. $-2(10)^{x+4} = -.002$

$10^{x+4} = .001$
$10^{x+4} = 10^{-3}$
$x+4 = -3$
$x = -7$

9. $(10)^x = 1$

$x = 0$

10. $3(10)^{x+4} + 3 = 15$

$3(10)^{x+4} = 12$
$10^{x+4} = 4$
$\log_{10} 4 = x+4$
$.6021 = x+4$
$-3.3979 = x$

11. $\frac{2}{3} \cdot \frac{3}{2}(10)^{x+2} = 1{,}500 \cdot \frac{2}{3}$

$10^{x+2} = 1000$
$10^{x+2} = 10^3$
$x+2 = 3$
$x = 1$

12. $\log_5(x-14) = 4$

$5^4 = x-14$
$639 = x$

13. $\log_3 x = 4$

$3^4 = x$
$81 = x$

14. $\log_{10} x = 3$

$10^3 = x$
$1000 = x$

15. $\log_4(x+3) + 2 = 4$

$\log_4(x+3) = 2$
$4^2 = x+3$
$16 = x+3$
$13 = x$

13. The population of the United States in 2006 was about 300 million and growing exponentially at a rate of about 0.7% per year. If that growth rate continues, the population of our country in year 2006 + t will be given by the function $P(t) = 300(10^{0.003t})$.

a. Explain how you can be sure that P(0)=300. t=0 is the initial population of 300 million

b. When is the U.S. population predicted to reach 400 million? $y = 300(10^{.003x})$

42 years is 400.96 million

14. The function $y = 14{,}000(0.8)^x$ represents the value of a car x years after purchase.

look at x=4 in table

a. Find how much the car will be worth in four years. \$5734.40

b. When will the car be worth \$7,000? ≈ 3.1 years

15

p. $-5(10)^{x-9} = -5{,}000$

q. $\frac{1}{2}(10)^{2x} = 50{,}000$

r. $10^{2x} = .0001$

s. $\log(x+5) = 2$

t. $\log_3(4x-3) = 4$

u. $\log_x 8 = 3$

v. $\log_x 144 = 2$

w. $\log_4(4x) = 3$

x. $\log(25x) = 2$

3. Find the inverse of each function

a. $y = \frac{1}{2}x - 5$

b. $y = 4x^2$

c. $y = \sqrt[3]{x+4}$

4. Rewrite each function in exponential form. (2 points each)

a. $216 = 6^x$

b. $x = 12^6$

c. $81 = 3^{8x}$

5. Rewrite each function in logarithmic form. (2 points each)

a. $log_3 243 = x$

b. $log_{15} x = 3$

d. $log_x 120 = 3$

6. Suppose that 500 mg of a medicine enters a hospital patient's bloodstream at noon and decays exponentially at a rate of 15% per hour. The exponential function $D(t) = 500(10^{-0.07t})$ models the amount of medicine active in the patient's blood at a time t hours later, where t is time in hours. Round answers to the nearest hundredth.

a. Find D(0).

b. Find D(3).

c. Use logarithms to determine when there is 150 mg of medicine in the patient's blood stream.

d. Use logarithms to determine when there is 10 mg of medicine in the patient's blood stream.

7. The function $y = 12{,}800\ (1.045)^x$ represents the value of a piece of artwork x years after purchase.

a. How much will the artwork be worth in 15 years?

16

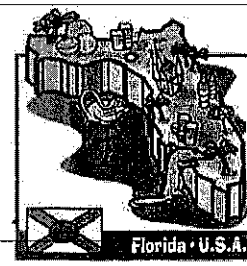
Common Core Math II Name Date

Independent Practice : Exponential Growth & Decay

1 POPULATION

In 1990, Florida's population was about 13 million. Since1990, the state's population has grown about 1.7% each year. This means that Florida's population is growing exponentially.

Year	Population
1990	13 mil
1991	13.221 mil
1992	13.446 mil
1993	13.674 mil
1994	13.907 mil



a) Write an explicit function in the form $y = ab^x$ that models the values in the table.

$y = 13(1.017)^x$

b) What does x represent in your function?

years since 1990

c) What is the "a" value in the equation and what does it represent in this context?

13 million people in Florida in 1990

d) What is the "b" value in the equation and what does it represent in this context?

1.017 = 100% + 1.7% → Florida keeps 100% of its people & grows 1.7%

2 HEALTHCARE

Since 1985, the daily cost of patient care in community hospitals in the United States has increased about 8.1% per year. In 1985, such hospital costs were an average of \$460 per day.

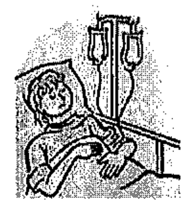
a) Write an equation to model the cost of hospital care. Let x = the number of years after 1985.

$y = 460(1.081)^x$

b) Find the approximate cost per day in 2012.

$x = 27$ $y = \$3767.49$

c) When will the cost per day be \$1000



x = 9.97 years y = \$1000 So at the end of 1994.

d) When will the cost per day be \$2000?

x = 18.869 years, y = \$2000 so at the end of 2003

3 HALF-LIFE

To treat some forms of cancer, doctors use Iodine-131 which has a half-life of 8 days. If a patient received 12 millicuries of Iodine-131, how much of the substance will remain in the patient 2 weeks later?

$y = 12(.5)^{\frac{14}{8}}$ = 3.57 millicuries

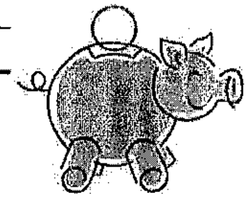
$y = a(b)^{\frac{x}{h}}$

$\uparrow$

$\frac{1}{2}$

17

4 SAVINGS



Suppose your parents deposited \$1500 in an account paying 6.5% interest compounded annually when you were born.

a) Find the account balance after 18 years.

$y = 1500(1.065)^{18} = \$4659.98$

b) What would be the difference in the balance after 18 years if the interest rate in the original problem was 8% instead of 6.5%?

$y = 1500(1.08)^{18} = \$5994.03$

5994.03 − 4659.98 = \$1334.05

you'd make \$1334.05 more with 8% interest.

c) What if the interest was 6.5% and was compounded monthly instead of annually.

$y = 1500\left(1 + \frac{.065}{12}\right)^{12 \cdot 18} = \$4817.75$

4817.75 − 4659.98 = \$157.77

you'd make \$157.77 more with interest compounded monthly.

5 HEALTH

Since 1980, the number of gallons of whole milk each person in the US drinks in a year has decreased 4.1% each year. In1980, each person drank an average of 16.5 gallons of whole milk per year.

Year	Population
1980	16.5
1981	17.2
1982	17.9
1983	18.6
1984	19.4

a) Write a recursive function for the data in the table.

NEXT = NOW · 1.041 start at 16.5

b) Write an explicit function in the form $y = ab^x$ that models the values in the table. Define your variables.

$y = 16.5(1.041)^x$

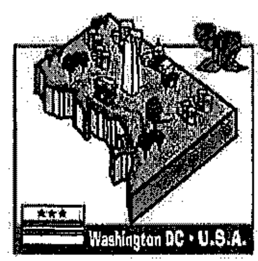
x = years after 1980

y = gallons of milk consumed each year.

c) According to this same trend, how many gallons of milk did a person drink in a year in 1970?

$x = -10$ $y = 11.04$ gallons

6 WASHINGTON, D.C.



The model $y = 604000(0.982)^x$ represents the population in Washington, D.C. x years after 1990.

a) How many people were there in 1990?

604000

b) What percentage growth or decay does this model imply?

1.8% decay

c) Write a recursive function to represent the same model as the provided explicit function.

NEXT = NOW · 0.982 start at 604000

d) Suppose the current trend continues, predict the number of people in DC now.

X = 23 (2013) y = 397741 people

e) Suppose the current trend continues, when will the population of DC be approximately half what it was in 1990?

$302000 = 604000(.982)^x$

$\frac{1}{2} = .982^x$

y_1 y_2

2nd trace #5

x = 38.2 years

In 2028, the population will be 302000.

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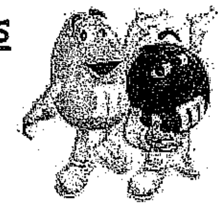
Adapted from Prentice Hall Algebra I pg 437-444

Name: Period: Date:

Math Lab: Modeling Cancer Cells with M&M's



Part I: Modeling Exponential Growth M&M Activity

The purpose of this lab is to provide a simple model to illustrate exponential growth of cancerous cells.

In our experiment, an M&M represents a cancerous cell. If the M&M lands "M" up, the cell divides into the "parent" cell and "daughter" cell. The cancerous cells divide like this uncontrollably-without end.

*We will conduct **7 trials** and record the number of "cancerous cells" on the plate.*

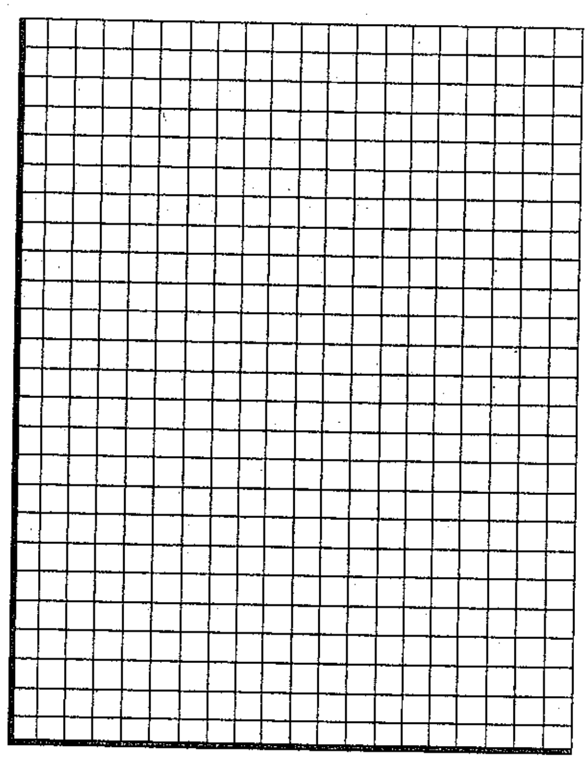
DO NOT EAT THE M&M's UNTIL YOU ARE DONE COLLECTING ALL DATA

Exponential Growth Procedure

1) Place 2 M&M's in a cup. This is trial number 0.
2) Shake the cup and dump out the M&Ms onto the paper plate. For every M&M with the "M" showing, add another M&M and then record the new population. (Ex. If 5 M&Ms land face up, then you add 5 more M&Ms)
3) Repeat step number 2 until you are done with 12 trials OR you run out of M&Ms.

Trial #	0	1	2	3	4	5	6	7	8	9	10	11	12
# of M&M's (# of cells)	2												

4) Graph your data (scatterplot) with the trial number on the x-axis and the number of M&M's on the y-axis. Label your scale.



5) Should your graph touch the x-axis? Why or why not?

6) Should your graph be just individual points, or should you connect the points? Explain. (Hint: What would an x-value of 3.5 mean in the context of the problem?

7) We can also use a graphing calculator to write the exponential growth equation.
You will need to enter your data table from page 1 into your graphing calculator. Click STAT, and under EDIT choose Edit. A blank table should appear. Under L_1 you are going to list the trial number and under L_2 list the Number of M&Ms. *(ONLY IF YOUR ALREADY HAVE DATA IN THE LISTS: To clear the lists before you begin, highlight the list name all the way at the top and press CLEAR—not delete—and ENTER)* Now you need to find the "curve of best fit". This will make an equation that best models your data. Go to your home screen (2nd QUIT), click STAT, scroll right to CALC, select ExpReg, press ENTER.
Write the exponential regression equation to three decimal places.

$y = \underset{a}{\underline{\hspace{3cm}}} * (\underset{b}{\underline{\hspace{3cm}}})^x$

8) Use your exponential growth model that you created in #7 to predict the number of "cancerous cells" there would be in:

Trial 25 ____________________ Trial 50 ________________________

9) Use your exponential growth model to determine the number of trials needed to have a population of 1 billion "cancerous cells". Show your work.

10) Why do we all have different values for a and b?

11) What do a and b represent in the context of the problem?

12) What would the "perfect" values for a and b be?

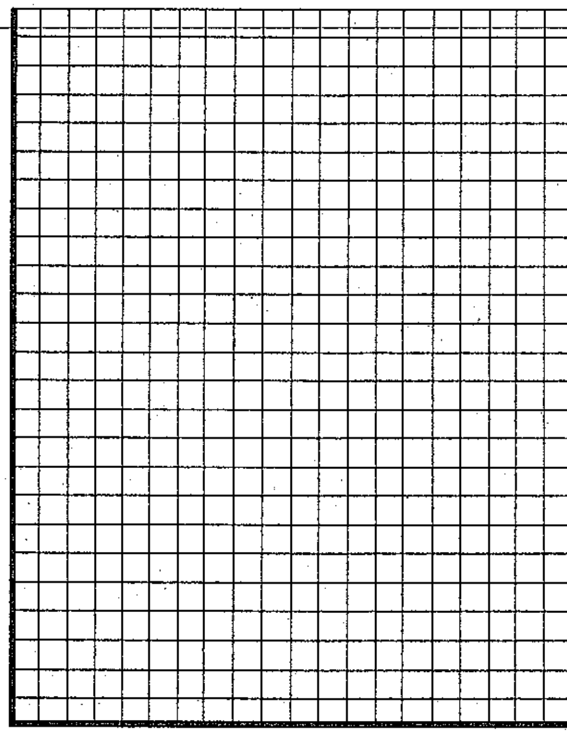
Part II: Modeling Exponential Decay

Exponential Decay Procedure

13) Count the total number of M&Ms that you have. Record this number in trial # 0.
14) This time when you shake the cup and dump out the M&Ms onto the plate, remove the M&Ms with the "M" showing. Record the M&M population.
15) Continue this process and fill in the table. You are done when you have completed 7 phases –OR– when your M&M population gets to 0. **Do NOT record 0 as the population, leave it blank!!!**

Trial #	0	1	2	3	4	5	6	7
M&M Population								

16) Sketch the graph representing your data. Label your scale.



Exponential Decay Discussion

17) In the instructions for #15 (in Part II), why do you think you are NOT supposed to reduce the number of M&Ms all the way to zero? Explain.

18) Using your calculator again, write the exponential regression equation to three decimal places

$$y = \underset{a}{\underline{\hspace{3cm}}} * (\underset{b}{\underline{\hspace{3cm}}})^x$$

19) Use the exponential decay model you found to determine your M&M population on the 4th trial?

20) How does this number compare to your actual data for the 4th trial. Are they the same? Are they similar? What are some reasons why your results are different? Explain.

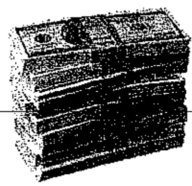
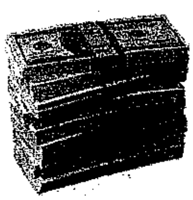
21) What would the "perfect" model be for this situation?

22) How could we change the way we gather the data to get our model closer to the "perfect" model?

Day 5

Exponential Growth & Decay Classwork

Name: ____________________

Question	Exponential Growth or Decay?	Write a function that represents this situation	Answer:
1. You buy a house for $130,000. It appreciates 6% per year. How much is it worth in 10 years? 	growth	**Initial Amount =** 130,000 **Growth/Decay Rate:** Percent = 6% Decimal = .06 Function that represents this situation: $y = 130{,}000(1+.06)^x$	$130{,}000(1.06)^{10}$ \$232,810.20
2. Justin Beiber is losing 20% of his hair each year. If he currently has 1,546 hairs on his head, about how many hairs will he have left after 10 years? 	decay	**Initial Amount =** 1546 **Growth/Decay Rate:** Percent = 20 Decimal = .20 Function that represents this situation: $y = 1546(1-.20)^x$	$1546(.8)^{10}$ 166 hairs
3. If you invest $40 in an account for 10 years at a 3% interest rate compounded semi-annually, how much money will you have? 	growth	**Initial Amount =** \$40 **Growth/Decay Rate:** Percent = 3% Decimal = .03 Function that represents this situation: $y = 40\left(1+\frac{.03}{2}\right)^{2x}$	$40(1.015)^{20}$ \$53.87
4. A population of 100 frogs increases at an annual rate of 22%. How many frogs will there be in 5 years? 	growth	**Initial Amount =** 100 frogs **Growth/Decay Rate:** Percent = 22% Decimal = .22	$100(1.22)^5$ ≈ 270 frogs

$y = 100(1+.22)^x$

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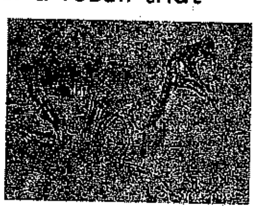
Exponential Growth & Decay Classwork

Name: ____________________

		Function that represents this situation:	
5. A species of extremely rare, deep water fish are slowly becoming extinct. If there are a total 821 of this type of fish and there are 15% fewer fish each month, how many will there be in half a year? 6 months 	decay	**Initial Amount =** 821 **Growth/Decay Rate:** Percent = 15% Decimal = .15 Function that represents this situation: $y = 821(1-.15)^x$	$821(.85)^6$ ≈ 309 fish
6. The population of Austin is growing at a rate of 5% per year. In 2010, the population was 500,000. What would be the predicted current population? 	growth	$y = 500{,}000(1+.05)^x$ 2017 $x=7$	$500000(1.05)^7$ 703,550 people
7. Use the equation from the previous question and predict in what year Austin's population will first reach 1,000,000.		$y = 500{,}000(1.05)^x$ Use table	15 years
8. Carbon-14 has a half-life of 5,730 years. If a fossil that originally had 500 mg of carbon-14 is found and determined to be 27,000 years old, how much carbon-14 was left? 		$y = 500\left(\frac{1}{2}\right)^{\frac{x}{5730}}$ $y = 500\left(\frac{1}{2}\right)^{\frac{27000}{5730}}$	19.08 mg
9. A super-deadly strain of bacteria is causing the zombie population to double every 2 days. Currently, there are 25 zombies. 	growth	$y = 25(2)^{\frac{x}{2}}$ use table	20 days

After how many days will there be 25,600 zombies?

Exponential Growth & Decay Notes/Review   Name: ______________

GROWTH Scenario: There has been a zombie invasion. The number of zombies increases by 45% each hour. If 3 zombies initially rolled into town at midnight, how many zombies will there be by 30 minutes past noon?

Step 1: Create a table for the scenario. Start with x=0

X	0	1	2	3	4	5	6	7	8	9	10	11	12	12.5 12:30	1
Y	3	840	6	9	13	19	28	40	59	85	123	178	259	312	376

Step 2: Write a recursive (NOW-NEXT) equation for the scenario:
Note: *The common ratio is the PERCENTAGE (written as a decimal) remaining after one time period has gone by.*

NEXT = NOW * (1.45)

Step 3: Write an explicit equation for the scenario:
Note: *All exponential equations are in the form $y=a*b^x$. a = initial value, b = common ratio*

$y = 3(1+.45)^x$

Step 4: x = the amount of time (or time periods) that have gone by. Choose/substitute an x in order to solve the question.

12.5

$3(1.45)^{12.5}$

Answer: 312 zombies

DECAY scenario: The zombie invasion is wiping out the population. The number of normal people are diminishing fast. Each day that goes by 48% of the living population is lost. If the population of North Carolina started out at 9.752 million, how many people will be left after one week?

Step 1: Create a table for the scenario. Start with x=0

X	0	1	2	3	4	5	6	7		
Y	9.752 million	5.071	.71303	.02711						

Step 2: Write a recursive (NOW-NEXT) equation for the scenario:
Note: *The common ratio is the PERCENTAGE (written as a decimal) remaining after one time period has gone by.*

(1 − .48)

Step 3: Write an explicit equation for the scenario:
Note: *All exponential equations are in the form $y=a*b^x$. a = initial value, b = common ratio*

$y = 9.752(1-.48)^x$

Step 4: x = the amount of time (or time periods) that have gone by. Choose/substitute an x in order to solve the question.

$x=7$

.100257 million

Answer: 100,257 people

> Note:
> - For growth scenarios → the common ratio is greater than 1. → Can be found by doing 100% + (% of increase) then write it as a decimal.
> - For decay scenarios → the common ratio is less than 1. → Can be written as 100% - (% of decrease) then write it as a decimal.

Special Circumstances

Compound Interest Scenario: Mary places \$5000 into a savings account that earns 3.1% interest compounded quarterly. How much money will Mary have in her account after 15 years?

> *NOTE: Compound Interest is a special type of GROWTH scenario. To calculate the common ratio: 1 + (% interest written as a decimal / # of times compounded per year)
> Additionally, x (amount of time) must be multiplied by the # of times compounded per year.
> Therefore, your final equation looks like:
>
> $$y = a(1+r/n)^{nx}$$
>
> where a = intial amount, r = interest rate as a decimal, n = number of times compounded per year, and x = amount of time
>
> | | | |
> |---|---|---|
> | Annually = 1 | Quarterly = 4 | Daily= 365 |
> | Semi-Annually = 2 | Weekly = 52 | |

$$y=5000\left(1+\frac{.031}{4}\right)^{4\cdot 15}$$

Answer: \$7,945.81

Half-Life Scenario: Actinium-226 has a half-life of 29 hours. If 100 mg of Actinium-226 disintegrates of a period of 72.5 hours, how many milligrams will remain?

***NOTE: Half-Life is a special DECAY scenario where your common ratio is ½ (because there is ½ remaining). X represents the NUMBER OF HALF-LIFE TIME PERIODS. Be careful with this!**

$$y= a\cdot\left(\frac{1}{2}\right)^{\frac{x}{h}}$$

$$y=100\left(\frac{1}{2}\right)^{\frac{x}{29}}$$

$$y=100\left(\frac{1}{2}\right)^{\frac{72.5}{29}}$$

$$y=17.68$$

Answer: 17.68 mg

Exponential Formulas

increasing
appreciating

Growth: $y = P(1 + r)^t$

Final Amount

decreasing, depreciating

Decay: $y = P(1 - r)^t$

rate

Principal or initial amount

$ Compound Interest: $y = P\left(1 + \frac{r}{n}\right)^{nt}$

annually – 1
semiannually – 2
quarterly – 4
monthly = 12

time

$ Compounded Continuously: $y = Pe^{rt}$

Euler's Number

Compounded n times

Half-Life: $y = P\left(\frac{1}{2}\right)^{t/h}$

length of half-life

27

Honors Common Core III

Applications of Exponential/Logs

Name:________________________

Date:________________________

Find the value of each investment after "t" years in interest is compounded continuously at the given annual rate "r" on the principal "P". Show your work! $A=Pe^{rt}$

1. t=2 years, r=7%, P=$6000

$A= 6000e^{.07(2)}$

$\boxed{A=\$6901.64}$

2. t=2.5 years, r=6%, P=$7500

$A= 7500e^{.06(2.5)}$

$\boxed{A=\$8713.76}$

3. t=5 years, r=3%, P=$5000

$A= 5000e^{.03(5)}$

$\boxed{A=\$5809.17}$

4. Find the "P" principal for the following:

t=3.5 years, r=4.5%, A=$10,000

$A=Pe^{rt}$

$\frac{10000}{e^{.045\times3.5}}=\frac{Pe^{.045(3.5)}}{e^{.045\times3.5}}$

$\boxed{\$8542.77}$

5. You have inherited an emerald ring that had an appraised value of $2400 in 1971. It is now 2007 and the appraised value of the ring has increased by approximately 6% each year.

2007−1971=36

a. What is the ring's current value?

$A=2400(1+.06)^{36}=\boxed{\$19,553.40}$

b. How long would it take for the ring to reach a value of $35,000?

$35,000=2400(1+.06)^{t}$

$\boxed{t=45.99 \text{ years}}$

6. $8000 in invested at 10% and compounded continuously.

a. How much money is there after 3 years?

$A=8000e^{.1(3)}=\boxed{\$10,798.87}$

b. How long will it take to for the account to quadruple?

$\ln 4 = \cancel{\ln} e^{.1t}$

$\ln 4 = .1t$

$\boxed{t=13.86 \text{ years}}$

28

7. Heather received \$100 for her thirteenth birthday. She has decided to save it in a bank with 5% interest compounded quarterly. $A = P\left(1+\frac{r}{n}\right)^{nt}$

a. How much money will she have in the bank by her eighteenth birthday? \$128.20

$A = 100\left(1+\frac{.05}{4}\right)^{4(5)}$ =

b. When will the account reach a balance of \$300?

$300 = 100\left(1+\frac{.05}{4}\right)^{4t}$

$3 = \left(1+\frac{.05}{4}\right)^{4t}$

$3 = 1.0125^{4t}$

$\frac{\log_{1.0125} 3}{4} = \frac{4t}{4}$

$22.11 = t$ years

8. There are 10 grams of Curium-245 which has a half-life of 9,300 years. How many grams will remain after 37,200 years?

$A = 10\left(\frac{1}{2}\right)^{\frac{37200}{9300}} = .625$ grams

9. A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take half the caffeine to be eliminated from a person's body?

Graph y_1, y_2
Calc, Intersection
Enter thrice

$y = a(1 \pm r)^t$

$y = 130(1-.11)^x$

$65 = 130(1-.11)^x$

$\frac{1}{2} = .89^x$

$\log_{.89} \frac{1}{2} = x$

$5.95 = x$ hours

10. You have deposited \$500 in an account that pays 6.75% interest, compounded continuously. How long will it take your money to triple?

$1500 = 500e^{.0675t}$

$3 = e^{.0675t}$

$\ln 3 = .0675t$

$16.28 = t$ years

11. The air pressure, P, at sea level is about 14.7 pounds per square inch. As the altitude increases the air pressure decreases. The relationship between air pressure and altitude can be modeled by the equation $P = 14.7e^{-0.00004h}$. Mount Everest rises to a height of 29,108 feet about sea level. What is the air pressure at the peak of Mt. Everest?

$P = 14.7e^{-.00004(29,108)}$

$P = 4.59$ pounds per square in

12. Roland earned \$1500 last summer. If he deposited the money in a certificate of deposit that earns 12.5% interest compounded monthly, how much money will he have next summer?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$= 1500\left(1+\frac{.125}{12}\right)^{12(1)}$

$A =$ \$1698.62

29

13. Carmen is saving for a new car which will cost $15,000. If she puts $5,000 in an account which earns 10% interest compounded monthly, how long will it take for her to save enough money to buy the car?

$15000 = 5000\left(1+\frac{.1}{12}\right)^{12t}$

$3 = \left(1+\frac{.1}{12}\right)^{12t}$

$\frac{\log_{(1+\frac{.1}{12})} 3}{12} = \frac{12t}{12}$

$t = 11.03$ years

14. Using carbon dating, scientists can determine how old a fossil is by how much Carbon-14 is present. If an average animal carcass contains 1 gram of Carbon-14 with a half-life of 5760 years, how old is a fossil with 0.0625 grams of Carbon-14?

$y = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$.0625 = 1\left(\frac{1}{2}\right)^{\frac{t}{5760}}$

$5760 \cdot \log_{\frac{1}{2}} .0625 = \frac{t}{5760} \cdot 5760$

$23040 = t$

15. A new car costs $23,000. It is expected to depreciate at a rate of 12% each year.

a. What will the value of the car be in 5 years? $y = 23000(1-.12)^5 = \boxed{\$12137.83}$

b. When will the car reach a value of $5000? $5000 = 23000(1-.12)^x$

$x = 11.9$ years

16. In 2000, the population of Phoenix was 1,321,045 and it increased to 1,331,391 in 2004. Suppose the population of Phoenix continues to increase at the same rate. What would be a good estimate for the population in 2015?

$1{,}331{,}391 = 1{,}321{,}045(1+r)^4$

$\sqrt[4]{1.00783} = \sqrt[4]{(1+r)^4}$

$1.00195 = 1 + r$
$-1 \qquad -1$

$.00195 = r$

$y = 1{,}321{,}045(1+.00195)^{15}$

$y = 1{,}360{,}262$

17. The Quadratics Creamery company has a savings plan for their employees. An employee can make an initial contribution of $2500 and the company will pay 7.5% interest compounded quarterly.

$A = P\left(1+\frac{r}{n}\right)^{nt}$

a. How much money will the employee have after 10 years? $A = 2500\left(1+\frac{.075}{4}\right)^{4(10)}$

$= 5255.87$

b. How long will it take an employee to earn $4000?

$y_1 = 2500\left(1+\frac{.075}{4}\right)^{4x}$

$y_2 = 4000$

$x = 6.33$

30

18. There are 80 grams of Cobalt-58 which has a half-life of 71 days. How many grams will remain after 213 days?

80
40 71
20 142
10 213

$y = 80(.5)^{\frac{213}{71}} = 10$

19. You purchased a Mac computer for \$3000 four years ago. It is now only worth \$800.

a. What is the rate at which it is depreciating?

$800 = 3000(1-r)^4$

$\frac{800}{3000} \quad \frac{3000}{3000}$

$\sqrt[4]{\frac{4}{15}} \quad \sqrt[4]{(1-r)^4}$

$.7186 = 1 - r$

$-.2814 = -r$

$.2814 = r$

28.14%

b. When was the computer be worth half its original cost?

$1500 = 3000(1-.2814)^x$

$.5 = .7186^x$

$\log_{.7186} .5 = x$

$x = 2.1$ years

20. Sarita deposits \$1000 in an account paying 3.4% annual interest compounded continuously.

a. What is the balance in the account after 5 years?

$A = Pe^{rt}$

$= 1000e^{.034(5)}$

$A = \$1185.30$

b. How long will it take the account to reach \$2000?

$2000 = 1000e^{.034x}$

$\ln 2 = \ln e^{.034x}$

$\ln 2 = .034x$

$x = 20.39$ years

21. Juan invests \$7500 at 7.75% interest for one year. How much money will he have if the interest was compounded...

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

a. yearly? $A = 7500\left(1 + \frac{.0775}{1}\right)^1$

b. daily? $A = 7500\left(1 + \frac{.0775}{365}\right)^{365 \cdot 1}$

$A = 8104.30$

$A = Pe^{rt}$

$A = 7500e^{.0775(1)}$

$A = 8104.37$

22. You will deposit \$500 into an account paying 3% annual interest compounded continuously.

a. What is the balance after 5 years? $A = 500e^{.03(5)} = 580.92$

b. How long will it take for the balance to reach \$1200?

$1200 = 500e^{.03x}$

$\ln 2.4 = \ln e^{.03x}$

$\frac{\ln 2.4}{.03} = \frac{.03x}{.03}$

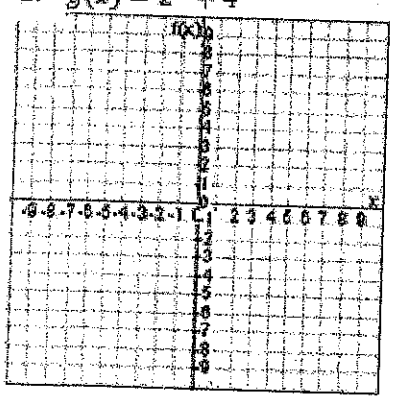
31 ( )

Transforming Exponential Functions

Translate left or right:	$g(x) = b^{x+c}$ (graph moves c units left) $g(x) = b^{x-c}$ (graph moves c units right)
Vertical stretch or compression:	$g(x) = cb^{x}$ (graph stretches if $c > 1$) (graph shrinks if $0 < c < 1$)
Horizontal stretch or compression:	$g(x) = b^{cx}$ (graph shrinks if $c > 1$) (graph stretches if $0 < c < 1$)
Reflections:	$g(x) = -b^{x}$ (graph reflects over the x-axis) $g(x) = b^{-x}$ (graph reflects over the y-axis)
Translate up or down:	$g(x) = b^{x} + c$ (graph moves up c units) $g(x) = b^{x} - c$ (graph moves down c units)

Part 3: Describe the transformation using the function f(x) = 2^x as the parent function. Then graph the function. For each, identify the domain, range, y-intercept, the asymptote, and the end behavior as $x \to \infty$ and $-\infty$. horizontal asymptote.

1. $g(x) = 2^x + 4$



Domain: ___________

Range: ___________

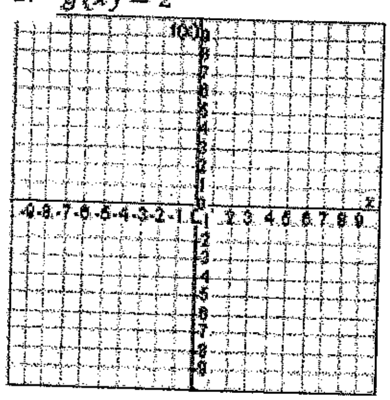
Y-Intercept: ___________

Asymptote: ___________

End Behavior: ___________

____

2. $g(x) = 2^{x+4}$



Domain: ___________

Range: ___________

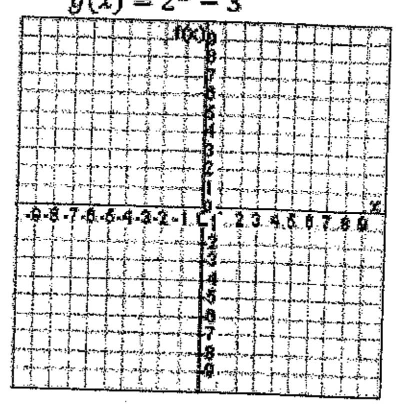
Y-Intercept: ___________

Asymptote: ___________

End Behavior: ___________

____

3. $g(x) = 2^x - 3$



Domain: ___________

Range: ___________

Y-Intercept: ___________

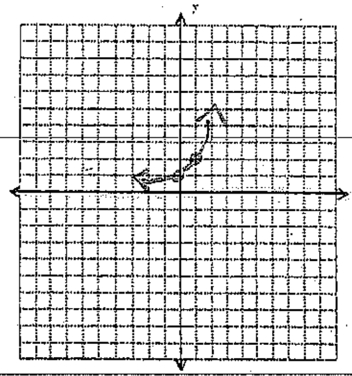
Asymptote: ___________

End Behavior: ___________

____

Graphing Exponential Functions Notes

Parent Graph: $y = b^x$



important information:

(0, 1)

(1, b)

y=0 horizontal asymptote

How do I transform exponential functions?

$$f(x) = a \cdot b^{x-h} + k$$

−a reflect x axis
0<a<1 compress
a>1 stretch

translate left/right

translate up/down → Horizontal Asymptote H.A.

How do I graph them?

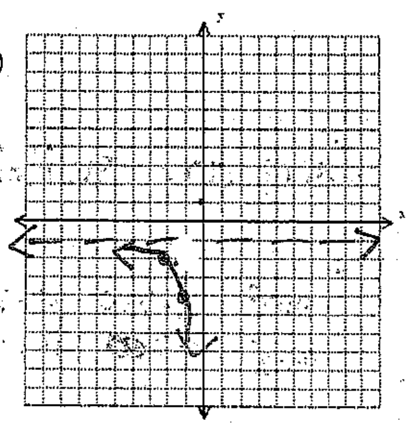
$y = -3^{x+2} - 1$ 2 1

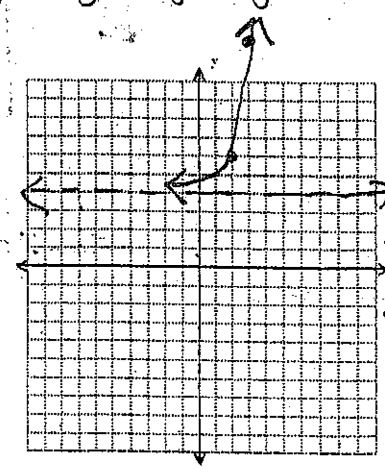
Parent	reflect	left	down	
(0,1)	(0,-1)	(-2,-1)	(-2,-2)	
(1,3)	(1,-3)	-1,-3	(-1,-4)	
H.A. y=0	y=0	y=0	y=-1	

$y = 2 \cdot 4^{x-2} + 4$ 2 4

	stretch	right	up	
(0,1)	(0,2)	(2,2)	(2,6)	
(1,4)	(1,8)	(3,8)	(3,12)	
H.A. y=0	y=0	y=0	y=4	

(−∞,∞)
(−∞,−1)
c never
c (−∞,∞)
t none
nt (0,−10)





D (−∞,∞)
R (4,∞)
Inc (−∞,∞)
Dec none
xint none
yint (0, 4.125)

Graphs of Exponential Functions

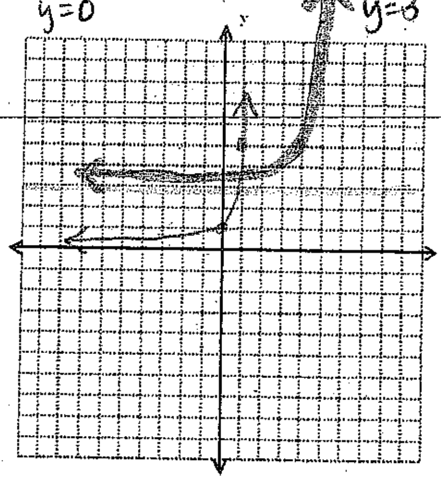
Identify the transformations for each of the following functions. Then draw a chart so that you can graph each equation.

1. $y = 2 \cdot 5^{x-4} + 3$ HA: y=3

vertical stretch x2
Right 4
Up 3

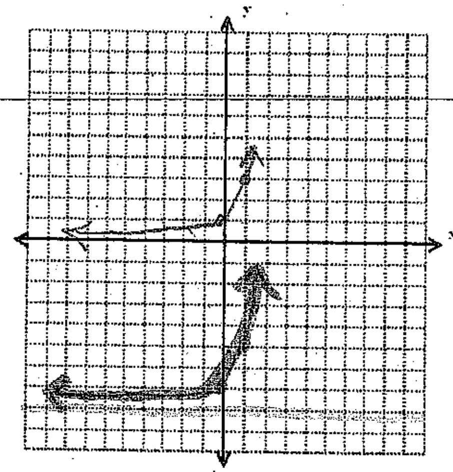
Parent	stretch	R4	Up3
(0,1)	(0,2)	(4,2)	(4,5)
(1,5)	(1,10)	(5,10)	(5,13)

HA: y=0 y=3



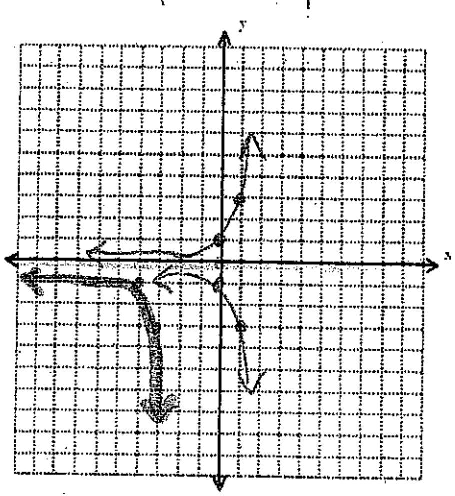
2. $f(x) = 3^x - 8$ HA: y=-8

Parent	Down 8
(0,1)	(0,-7)
(1,3)	(1,-5)



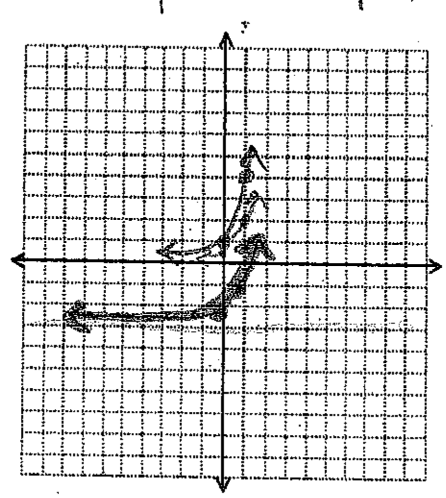
3. $y = -(3)^{x+4} + 0$ HA: y=0

Parent	Reflect X axis	Left 4
(0,1)	(0,-1)	-4,-1
(1,3)	(1,-3)	



4. $f(x) = \frac{1}{2}(4)^x - 3$ HA: y=-3

Parent	vertical stretch x1/2	Down 3
(0,1)	(0,1/2)	(0,-2 1/2)
(1,4)	(1,2)	(1,-1)

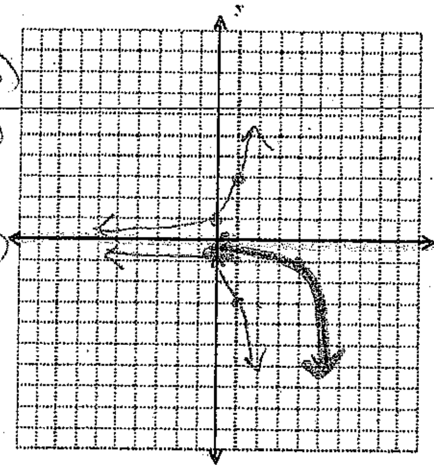


Graph. Identify the domain and range for each of the following functions. Then identify the increasing and decreasing intervals.

5. $y = -2 \cdot 3^{x-4}$

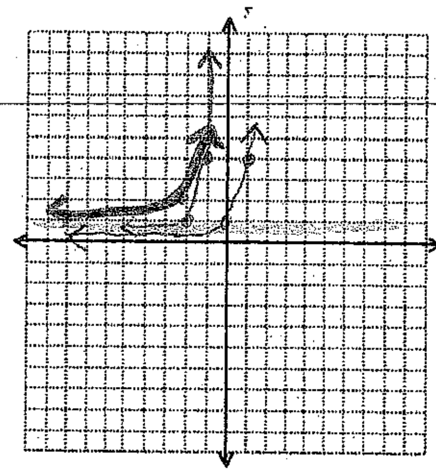
Parent	x axis Reflect	Right 4
(0,1)	(0,-1)	(4,-1)
(1,3)	(1,-3)	(5,-3)

: $(-\infty,\infty)$
: $(-\infty,0)$
c: none
c: $(-\infty,\infty)$



6. $f(x) = 4^{x+2} + 1$

Parent	Left 2	up 1
(0,1)	(-2,1)	(-2,2)
(1,4)	(-1,4)	(-1,5)

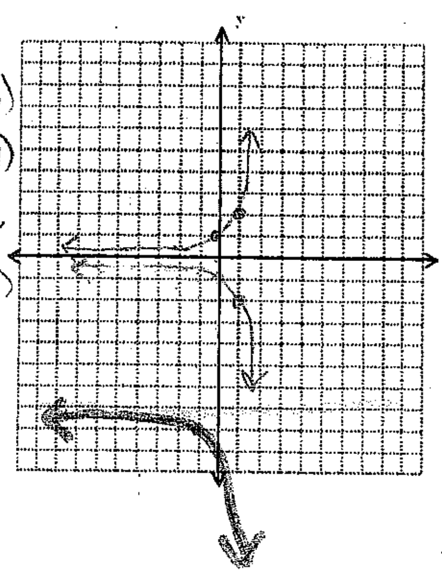


D: $(-\infty,\infty)$
R: $(1,\infty)$
Inc: $(-\infty,\infty)$
Dec: none

7. $y = -(2)^{x+1} - 7$

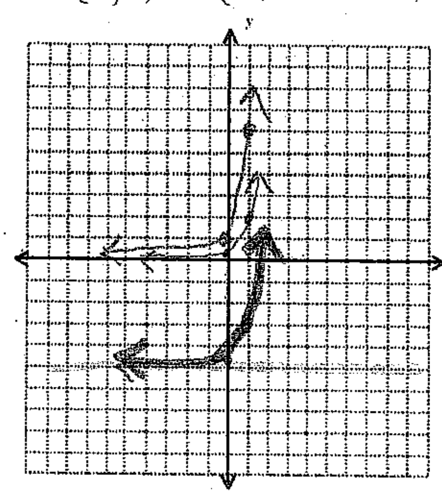
Parent	Reflect	Left 1	Down 7
(0,1)	(0,-1)	(-1,-1)	(-1,-8)
(1,2)	(1,-2)	(0,-2)	(0,-9)

: $(-\infty,\infty)$
: $(-\infty,-7)$
c: none
c: $(-\infty,\infty)$



8. $f(x) = \frac{1}{3}(6)^x - 5$

Parent	$\times \frac{1}{3}$ Compress	Down 5
(0,1)	(0, 1/3)	(0, -4 2/3)
(1,6)	(1,2)	(1, -3)



D: $(-\infty,\infty)$
R: $(-5,\infty)$
Inc: $(-\infty,\infty)$
Dec: none

15

2nd trace, zero, left bound, right bound, guess, enter enter enter

Graph the following on your calculator to find the x- and y-intercepts.

9. $f(x) = 5^{x+1} - 2$

x int: (-.57, 0)
y int: (0, 3)

10. $f(x) = \frac{1}{2} \cdot 3^x$

x int: none
y int: (0, ½)

Find the horizontal asymptote for each of the following functions. Then find the range.

11. $f(x) = -2^x - 7$

HA: y = -7
R: (-∞, -7)

12. $f(x) = \frac{1}{3}(2)^{x-3}$

HA: y = 0
R: (0, ∞)

Identify the base of the exponent. Then identify all of the transformations for each of the following functions.

13. $y = 2(5)^{x-4} + 3$

Base = 5
Vertical stretch x2, right 4, up 3

14. $y = -2(3)^x + 5$

Base = 3
Reflect x axis, up 5

15. $f(x) = -\frac{1}{3}^{x+6} - 3$

Base = $\frac{1}{3}$
Reflect x axis, left 6, down 3

16. $f(x) = 4^{x+1}$

Base = 4
Left 1

Find all of the following for $f(x) = -2(3)^{x+3} + 5$

17. Find the horizontal asymptote. y = 5

18. Find the domain and range.

D: -∞, ∞
R: (-∞, 5)

19. Find the increasing and decreasing intervals.

Inc: none
Dec: (-∞, ∞)

20. List all of the transformations.

Reflect x axis
Vertical stretch x 2
Left 3

96

Logarithmic Functions Notes

Name: ____________________

Date ____________________ Pd: ____

The logarithmic function is the inverse of the exponential function.

- $\log_a x = y$ is read "log base a of X equals y."
- It is equivalent to $a^y = x$

power

$\log_a x = y$ $a^y = x$

base

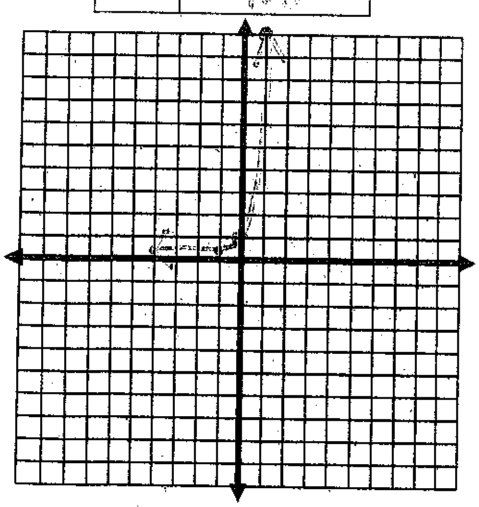
Practice: Change to the other form:

Exponential Form	$2^3 = 8$	$2^{-3} = \frac{1}{8}$	$7^m = x$	$10^3 = 1000$
Logarithmic Form	$\log_2 8 = 3$	$\log_2\left(\frac{1}{8}\right) = -3$	$\log_7 x = m$	$\log_{10} 1000 = 3$

Now let's use $f(x) = 10^x$ to explore its inverse, $f^{-1}(x) = \log_{10} x$

1. Complete the table to get the characteristic points of $f(x) = 10^x$ and then sketch the graph.

x	$f(x) = 10^x$
-1	$\frac{1}{10}$
0	1
1	10
2	100



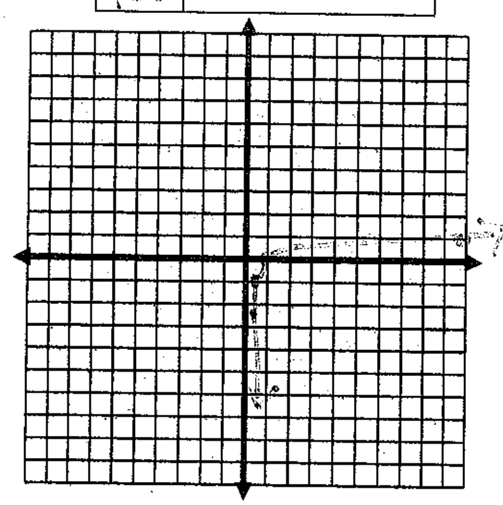
Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

2. Complete the table to get the characteristic points of $f^{-1}(x) = \log_{10} x$ and then sketch the graph.

x	$f^{-1}(x) = \log_{10} x$
$\frac{1}{10}$	-1
1	0
10	1
100	2



Asymptote: $x = 0$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

37

Logarithmic Functions Practice

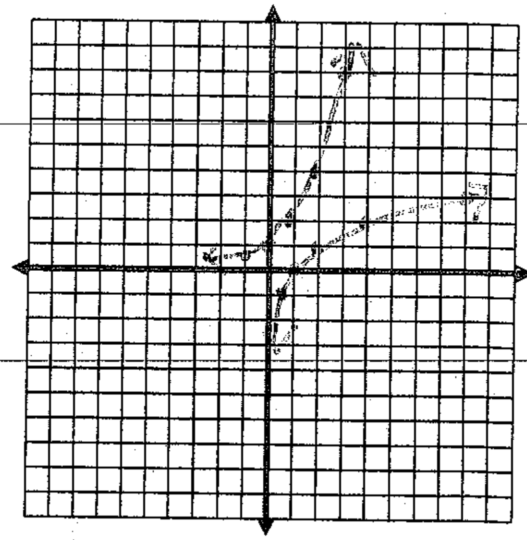
Name: ____________________
Date: ____________________ Pd: ____

1. Graph the exponential function and its inverse on the grid.

$y = 2^x$ and $y = \log_2 x$

x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

x	y
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3

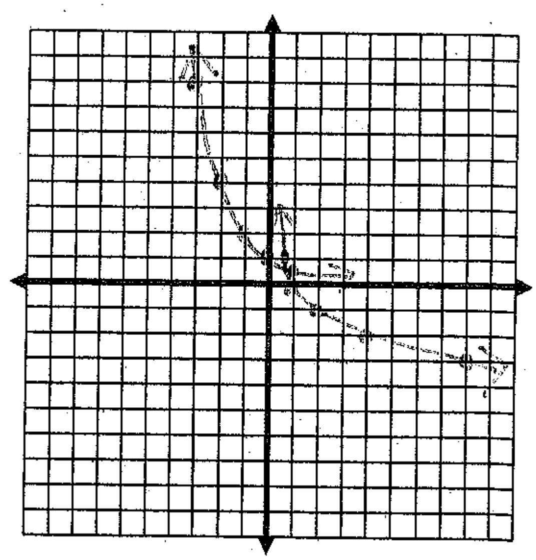


2. Graph the exponential function and its inverse on the grid.

$y = \left(\frac{1}{2}\right)^x$ and $y = \log_{\frac{1}{2}} x$

x	y
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$

x	y
8	-3
4	-2
2	-1
1	0
$\frac{1}{2}$	1



3. List the characteristic points of $y = \log_{10} x$. (10, 1) (1, 0)

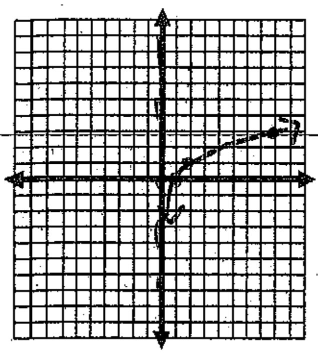
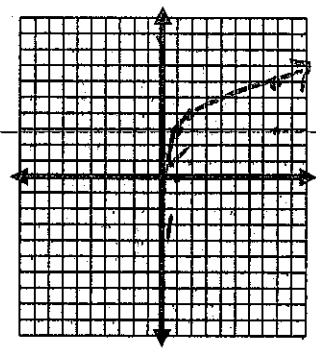
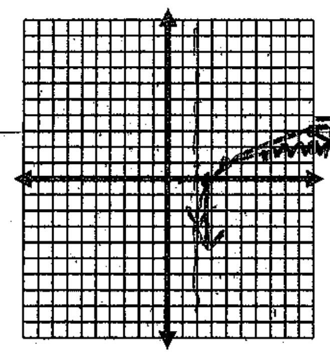
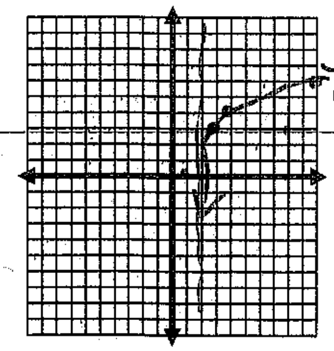
38

Logarithmic Functions, Practice

Name: ____________________
Date: ____________________ Pd: _____

1. Describe in your own words what happens to the graph of $f(x) = \log_2 x$ under the given transformations, then graph.

(1,0) (2,1) (8,3)

Base Graph: $f(x) = \log_2 x$	$f(x) = \log_2(x) + 3$	$f(x) = \log_2(x-2)$	$f(x) = \log_2(x-2) + 3$
Transformations: none	Transformations: up 3	Transformations: right 2	Transformations: right 2 up 3
			
Asymptote: $x=0$	Asymptote: $x=0$	Asymptote: $x=2$	Asymptote: $x=2$
Intercept(s): $(1,0)$	Intercept(s): $(\frac{1}{8},0)$	Intercept(s): $(3,0)$	Intercept(s): $(2\frac{1}{8},0)$
Domain: $(0,\infty)$	Domain: $(0,\infty)$	Domain: $(2,\infty)$	Domain: $(2,\infty)$
Range: $(-\infty,\infty)$	Range: $(-\infty,\infty)$	Range: $(-\infty,\infty)$	Range: $(-\infty,\infty)$

2. Describe the transformations of $y = 4\ \log_x(x-7) + 6$ from the parent function $f(x) = \log_2 x$.

right 7

up 6

dilate, vertical stretch ×4

3. Describe the transformations of $y = -\frac{1}{5}\log_{10}(x+3) - 2$ from the parent function $f(x) = \log_{10} x$.

down 2

reflect x axis

dilate vertical compression

left 3

$0 = \log_2(x) + 3$

$-3 = \log_2(x)$

$2^{-3} = x$

$\frac{1}{8} = x$

$0 = \log_2(x-2) + 3$

$-3 = \log_2(x-2)$

$2^{-3} = x - 2$

$+2 \quad +2$

$2 + \frac{1}{8} = x$

$2\frac{1}{8} = x$

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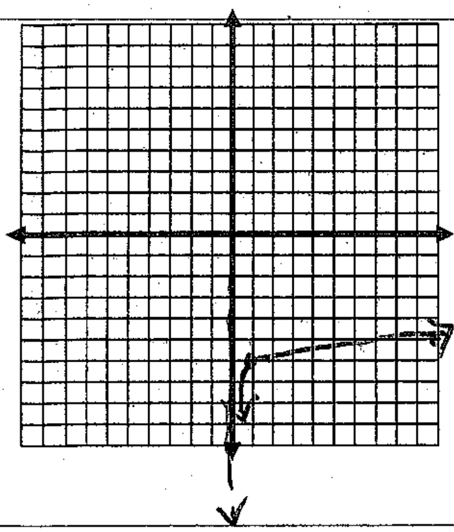
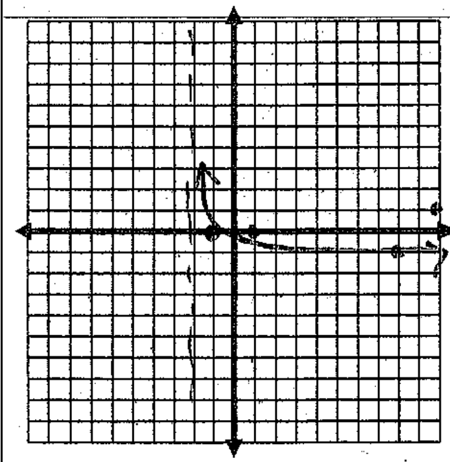
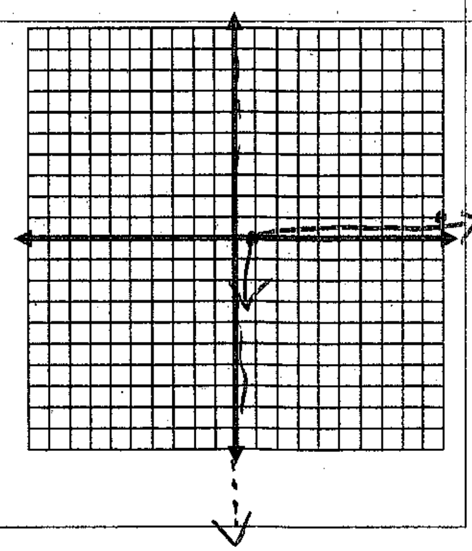
Parent $y = \log_{10} x$ (10,1) (1,0)

Determine the transformations as compared to the base graph, $y = \log_{10} x$. Graph each function on the coordinate planes provided. Determine the domain, range, and asymptotes of each transformation.

(1,-6) (10,-5)

4. $y = \log_{10} x - 6$	5. $y = -\log_{10}(x+2)$	6. $y = \frac{1}{2}\log_{10} x$
Transformations: down 6	Transformations: reflect x axis left 2	Transformations: vertical compression of 1/2
Asymptote: x=0	Asymptote: x=-2	Asymptote: x=0
Domain: (0,∞)	Domain: (-2,∞)	Domain: (0,∞)
Range: (-∞,∞)	Range: (-∞,∞)	Range: (-∞,∞)
		

Kuta Software - Infinite Precalculus Name________________________

Inverses of Logarithms

Date________________ Period____

Find the inverse of each function.

1) $y = 2\log_x 3$

$y = 3^{\frac{2}{x}}$

$x = 2\log_y 3$

$\sqrt[x]{y^x} = \sqrt[x]{3^2}$

$y = 3^{2/x}$

2) $y = \log_6 3^x$

$y = \log_3 6^x$

$x = \log_6 3^y$

$y = \frac{x}{\log_6 3}$

$6^x = 3^y$

$\log 6^x = \log 3^y$

$\log 6^x = y \cdot \log 3$

$\frac{\log 6^x}{\log 3} = y$

$\log_3 6^x = y$

or

$\frac{x}{\log_6 3} = \frac{y \cdot \log_6 3}{\log_6 3}$

3) $y = \log_2 x^3$

$y = 2^{\frac{x}{3}}$

$x = \log_2 y^3$

$\sqrt[3]{2^x} = \sqrt[3]{y^3}$

$y = 2^{x/3}$

4) $y = \log_5 (-2x)$

$y = -\frac{5^x}{2}$

$x = \log_5(-2y)$

$\frac{5^x}{-2} = \frac{-2y}{-2}$

$y = -\frac{5x}{2}$

5) $y = \log_6 (3x)$

$y = \frac{6^x}{3}$

$x = \log_6(3y)$

$\frac{6^x}{3} = \frac{3y}{3}$

$\frac{6^x}{3} = y$

6) $y = \log_4 x + 10$

$y = 4^{x-10}$

$x = \log_4 y + 10$

$x - 10 = \log_4 y$

$4^{x-10} = y$

7) $y = \log_2 x + 6$

$y = 2^{x-6}$

$x = \log_2 y + 6$

$x - 6 = \log_2 y$

$2^{x-6} = y$

8) $y = \log_6 x - 7$

$y = 6^{x+7}$

$x = \log_6 y - 7$

$x + 7 = \log_6 y$

$6^{x+7} = y$

9) $y = \log_x 2 - 6$

$y = 2^{\frac{1}{x+6}}$

$x = \log_y 2 - 6$

$x + 6 = \log_y 2$

$\sqrt[x+6]{y^{x+6}} = \sqrt[x+6]{2}$

$y = 2^{1/(x+6)}$

10) $y = 4\log_x 2$

$y = 2^{\frac{4}{x}}$

$x = 4\log_y 2$

$x = \log_y 2^4$

$\sqrt[x]{y^x} = \sqrt[x]{2^4}$

$y = 2^{4/x}$

11) $y = \log_5 (x + 5)$

$y = 5^x - 5$

$x = \log_5(y+5)$

$5^x = y + 5$

$5^x - 5 = y$

12) $y = -6\log_x 5$

$y = 5^{-\frac{6}{x}}$

$x = -6\log_y 5$

$x = \log_y 5^{-6}$

$\sqrt[x]{y^x} = \sqrt[x]{5^{-6}}$

$y = 5^{-6/x}$

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13) $y = 10^{\frac{x}{2}}$

$y = \log x^2$

$x = 10^{\frac{y}{2}}$

$\log_{10}$

$2 \cdot \log_{10} x = \frac{y}{2} \cdot 2$

$2 \log_{10} x = y$

$\log_{10} x^2 = y$

14) $y = 4^{\frac{x}{3}}$

$y = \log_4 x^3$

$x = 4^{\frac{y}{3}}$

$3 \cdot \log_4 x = \frac{y}{3} \cdot 3$

$3 \cdot \log_4 x = y$

$\log_4 x^3 = y$

15) $y = 3^x + 4$

$y = \log_3 (x - 4)$

$x = 3^y + 4$

$x - 4 = 3^y$

$\log_3 (x-4) = y$

16) $y = x$

$y = x$

$x = y$

17) $y = 4^{\frac{x}{2}}$

$y = \log_4 x^2$

$x = 4^{\frac{y}{2}}$

$2 \cdot \log_4 x = \frac{y}{2} \cdot 2$

$\log_4 x^2 = y$

18) $y = 6^x + 2$

$y = \log_6 (x - 2)$

$x = 6^y + 2$

$x - 2 = 6^y$

$\log_6 (x-2) = y$

19) $y = \left(\dfrac{\left(\frac{1}{4}\right)^x - 5}{-2} \right)^{\frac{1}{3}}$

$y = \log_{\frac{1}{4}} (-2x^3 + 5)$

20) $y = \left(\dfrac{5^x - 9}{-3} \right)^{\frac{1}{2}}$

$y = \log_5 (-3x^2 + 9)$

21) $y = \left(\dfrac{4^x - 3}{4} \right)^{\frac{1}{4}}$

$y = \log_4 (4x^4 + 3)$

22) $y = \left(\dfrac{e^x - 10}{4} \right)^{\frac{1}{5}}$

$y = \ln (4x^5 + 10)$

23) $y = \left(\dfrac{6^x + 10}{-2} \right)^{\frac{1}{3}}$

$y = \log_6 (-2x^3 - 10)$

24) $y = \left(\dfrac{5^x + 8}{4} \right)^{\frac{1}{3}}$

$y = \log_5 (4x^3 - 8)$

Honors Math 3

Logarithms and Exponential Functions as Inverses

Find the inverse of each function.

1) $y = \log(-2x)$

2) $y = \log_2(x-4)$

3) $y = \ln x + 6$

4) $y = \log x + 2$

5) $y = -6\log_3 x$

6) $y = -3\log_5 x$

7) $y = \log_2 x^2$

8) $y = \log_2 x^4$

9) $y = \log_5 \dfrac{-4^x + 2}{2}$

10) $y = \dfrac{\sqrt[4]{8e^x + 8}}{2}$

11) $y = \log_4 \dfrac{10^x - 2}{2}$

12) $y = \log_2 \dfrac{-8 \cdot 5^x + 1}{-4 \cdot 5^x}$

13) $y = \dfrac{\sqrt[3]{-2 \cdot 6^x - 2}}{2}$

14) $y = \dfrac{\sqrt{2^x + 5}}{2}$

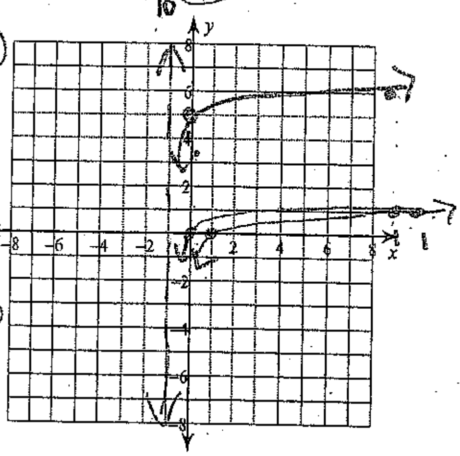
15) $y = \dfrac{\sqrt[5]{2 \cdot 3^{x+1} + 1}}{\sqrt[5]{4 \cdot 3^x}}$

16) $y = \dfrac{\sqrt[4]{-8 \cdot 10^x - 40}}{2}$

Identify the domain and range of each. Then sketch the graph.

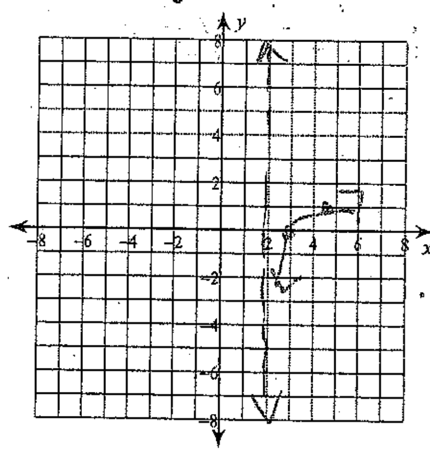
17) $f(x) = \log_{10}(x+1) + 5$ $x=-1$

(1,0) (10,1)
(0,0) (9,1)
(0,5) (9,6)
D: $(-1,\infty)$
R: $(-\infty,\infty)$



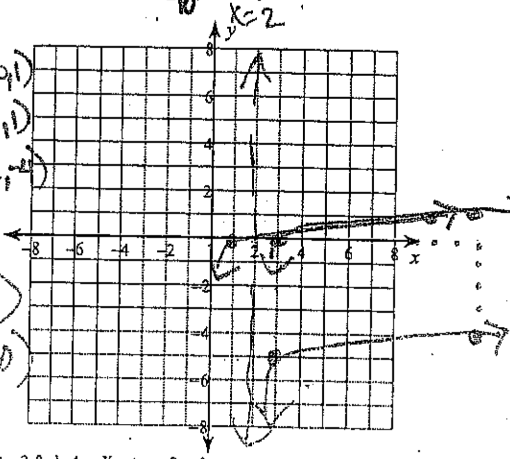
18) $f(x) = \ln_e(x-2)$ $x=2$

P (1,0) (e,1)
R2 (3,0) (e+2,1)
D: $(2,\infty)$
R: $(-\infty,\infty)$



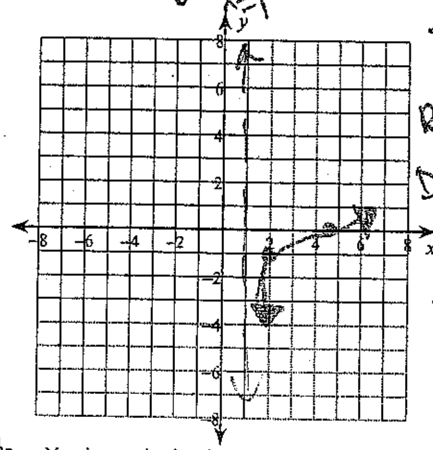
19) $f(x) = \log_{10}(x-2) - 5$ $x=2$

(1,0) (10,1)
(3,0) (12,1)
(3,-5) (12,-4)
D: $(2,\infty)$
R: $(-\infty,\infty)$



20) $f(x) = \ln_e(x-1) - 1$ $x=1$

P (1,0) (e,1)
R1 (2,0) (e+2,1)
D1 (2,-1) (e+2,0)
D: $(1,\infty)$
R: $(-\infty,\infty)$

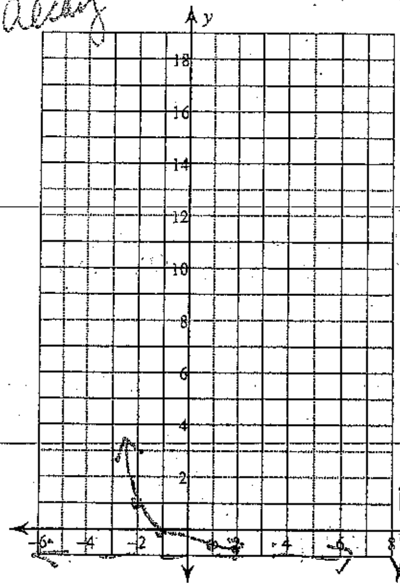


45. 23

Sketch the graph of each function.

21) $f(x) = \frac{1}{4} \cdot \left(\frac{1}{2}\right)^{x-1} - 1$ ↑ HA $y=-1$

decay



P = (0,1) (1, 1/2)
comp.
D 1/4 (0, 1/4) (1, 1/8)
R1 (1, 1/4) (2, 1/8)
D1 (1, −3/4) (2, −7/8)
D (−∞, ∞)
R (−1, ∞)
Inc never
Dec (−∞, ∞)
x int (−1, 0)
y int (0, −1/2)

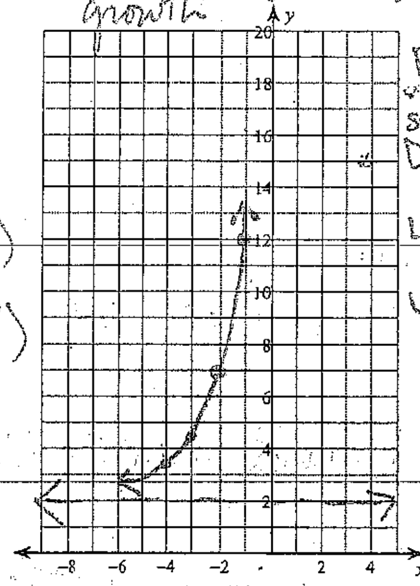
$y = \frac{1}{4}\left(\frac{1}{2}\right)^{0-1} - 1$

$= \frac{1}{4}(2) - 1$

$\frac{1}{2} - 1 = -\frac{1}{2}$

22) $f(x) = 5 \cdot 2^{x+2} + 2$ ↖ HA $y=2$

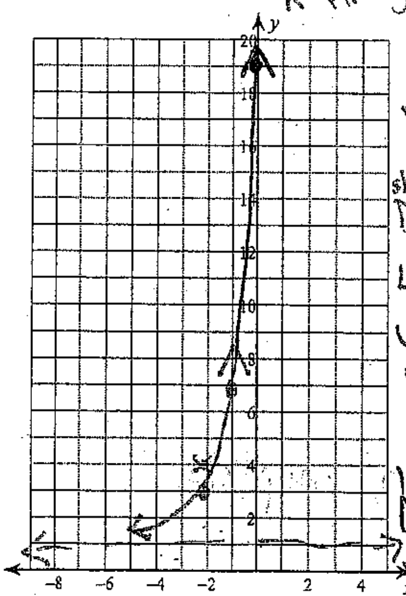
growth



P (0,1) (1, 2)
stretch
D5 (0,5) (1, 10)
L2 (−2,5) (−1, 10)
U2 (−2,7) (−1, 12)

growth

23) $f(x) = 2 \cdot 3^{x+2} + 1$ ↖ HA $y=1$



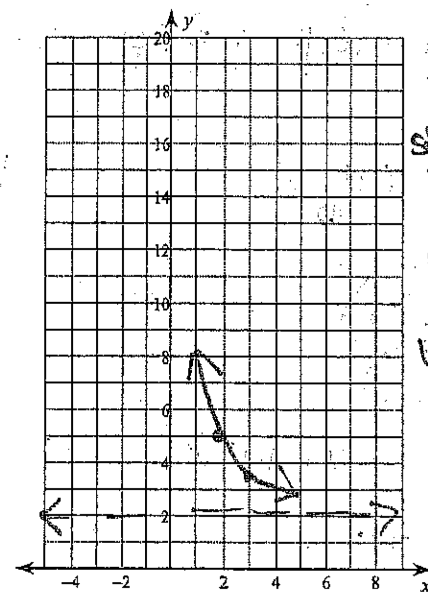
P (0,1) (1,3)
stretch
D2 (0,2) (1,6)
L2 (−2,2) (−1,6)
U1 (−2,3) (−1,7)
D (−∞, ∞)
R (1, ∞)
Inc (−∞, ∞)
Dec never
x int never
y int (0,19)

$y = 2 \cdot 3^{0+2} + 1$

$y = 19$

decay

24) $f(x) = 3 \cdot \left(\frac{1}{2}\right)^{x-2} + 2$ ← HA $y=2$



P (0,1) (1, 1/2)
stretch
D3 (0,3) (1, 3/2)
R2 (2,3) (3, 3/2)
U2 (2,5) (3, 7/2)

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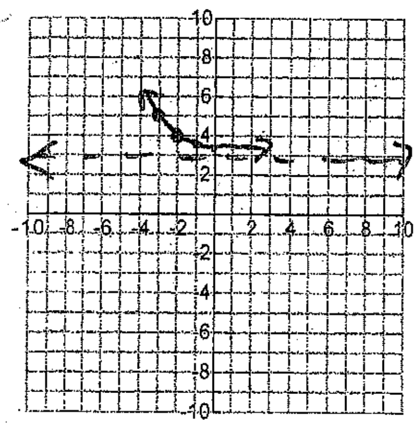
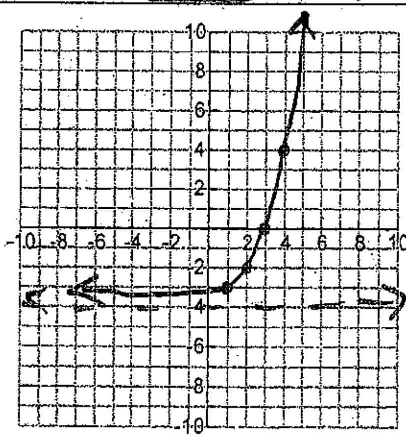
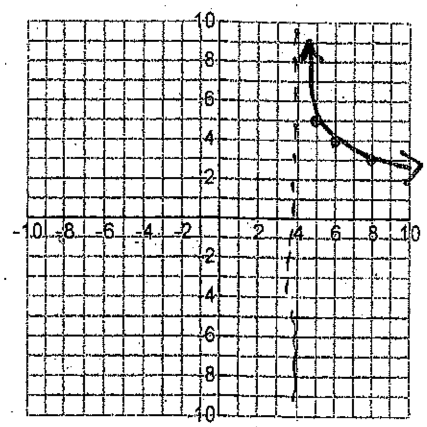
Unit Review Sheet

Name: ________________
Date: ____________ Period: _____

1. For each of the following functions, state the equation of the base function, the transformations from the base function, the domain, range, asymptotes. If the function is exponential, determine if it is a growth or decay model. Then graph each function.

a. $y = \left(\frac{1}{2}\right)^{x+2} + 3$	b. $y = (2)^{x-1} - 4$	c. $y = -log_2(x-4) + 5$
Base Function: $y=(\frac{1}{2})^x$	Base Function: $y = 2^x$	Base Function: $y = \log_2 x$
Transformations: left 2, up 3	Transformations: right 1, down 4	Transformations: reflect x axis, right 4, up 5
Domain: $(-\infty, \infty)$	Domain: $(-\infty, \infty)$	Domain: $(4, \infty)$
Range: $(3, \infty)$	Range: $(-4, \infty)$	Range: $(-\infty, \infty)$
Asymptote: $y=3$	Asymptote: $y=-4$	Asymptote: $x=4$
Circle One: **Growth** or **Decay**	Circle One: **Growth** or **Decay**	
		

2. Use the rules of exponents and/or logarithms to find the value of x in each equation. Round to the nearest hundredth when necessary.

a. $(3^{2x})(3^{12}) = 3^{20}$

$3^{2x+12} = 3^{20}$

$2x+12=20$

$2x=8$

$x=4$

b. $\frac{5^8}{5^{2x}} = 5^{10}$

$5^{8-2x} = 5^{10}$

$8-2x=10$

$-2=2x$

$-1=x$

c. $(13^4)^x = 13^{24}$

$13^{4x} = 13^{24}$

$4x=24$

$x=6$

d. $(25^{2x})(5^7) = 125^4$

$5^{2\cdot 2x} \cdot 5^7 = 5^{3\cdot 4}$

$5^{4x+7} = 5^{12}$

$4x+7=12$

$4x=5$

$x = 5/4$

e. $\frac{9^{5x}}{3^{2x}} = 81^{12} \rightarrow 3^{4\cdot 12}$

$\frac{3^{2\cdot 5x}}{3^{2x}} = \frac{3^{10x}}{3^{2x}} = 3^{8x} = 3^{48}$

$8x=48$

$x=6$

f. $(8^4)^x = 4^{18}$

$2^{3\cdot 4\cdot x}$ $2^{2\cdot 18}$

$2^{12x} = 2^{36}$

$12x=36$

$x=3$

g. $(49^{2x})(7^8) = 1$

$7^{2\cdot 2x} 7^8 = 7^0$

$7^{4x+8} = 7^0$

$4x+8=0$

$4x=-8$

$x=-2$

h. $(25)^{\frac{1}{2}}(3)^4 = x$

$5 \cdot 81 = x$

$405 = x$

i. $\left(6^{\frac{1}{2}}\right)\left(36^{\frac{3}{2}}\right) = 6^x$

$6^{-2} \cdot 6^{2\cdot 3/2} = 6^x$

$6^{-2} \cdot 6^3 = 6^x$

$6=6^x$

$1=x$

j. $10^{x+4} = 100{,}000{,}000$

$10^{x+4} = 10^8$

$x+4=8$

$x=4$

k. $6(10)^{5x} = 18{,}000$

$10^{5x} = 3000$

$\frac{\log_{10} 3000}{5} = \frac{5x}{5}$

$\boxed{.70 \approx .695 = x}$

l. $10^{3x-4} = 1{,}000$

$10^{3x-4} = 10^3$

$3x-4=3$

$3x=7$

$\boxed{x = 7/3}$

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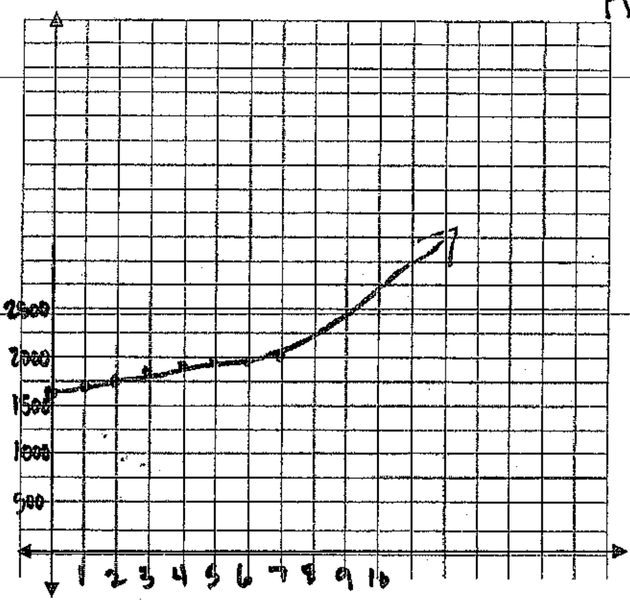
The principal amount of deposit is $1640. It has an interest rate of 3.2% compounded quarterly. Write a function and graph it. $A = 1640\left(1 + \frac{.032}{4}\right)^{4x}$

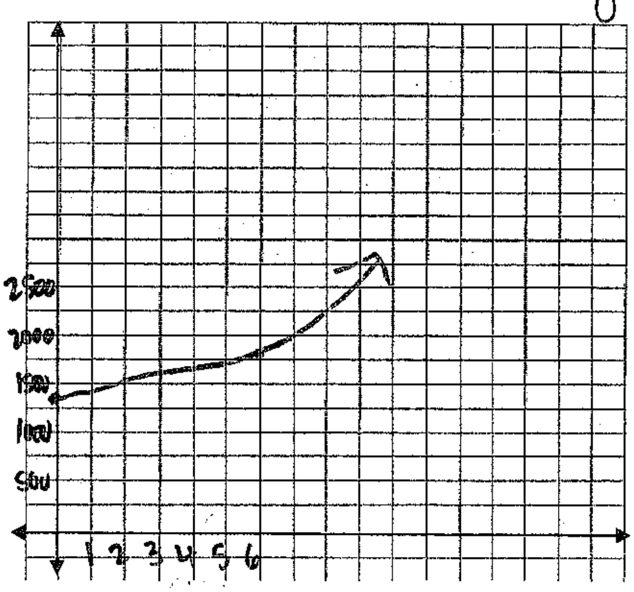
$A = 1640(1.008)^{4x}$ Graph + look @ table



From the graph, give an approximation of the balance after 3 years. \$1804.56

What is the balance to the nearest hundredth after 6 years? \$1985.62

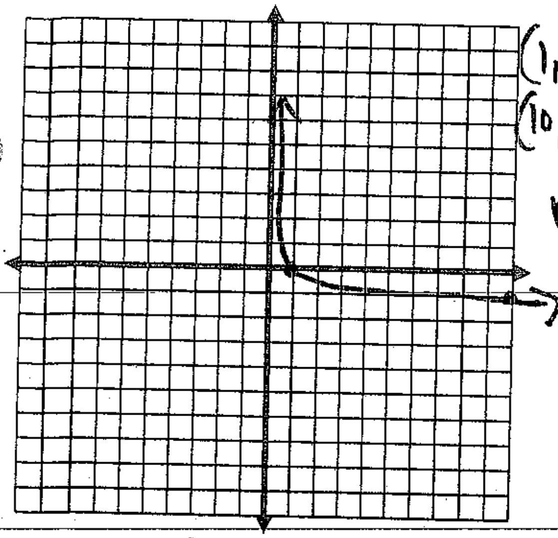
The principle amount of deposit is $1350. It has an interest rate of 4.6% compounded monthly. Write a function and graph it. $y = 1350\left(1 + \frac{.046}{12}\right)^{12x}$



From the graph, what is the approximate balance after 7 years?
1861.70

What is the balance to the nearest hundredth after 10 years?
\$2136.62

1. Graph $f(x) = -\log_{10} x$



(1,0)
(10,-1)

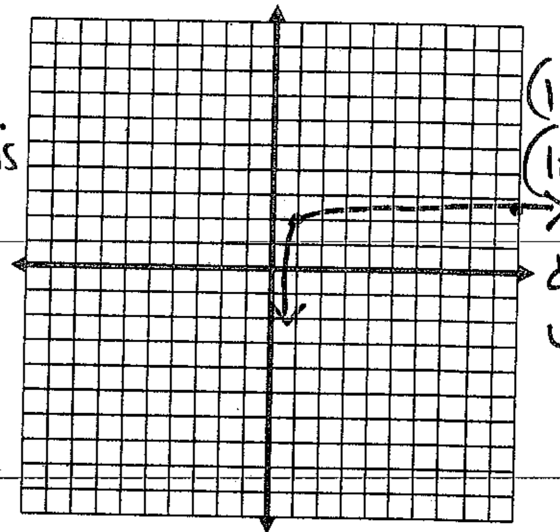
reflect x axis

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = 0$

3. Graph $f(x) = \left(\frac{1}{2}\right)\log_{10} x + 2$



(1,2)
(10,2.5)

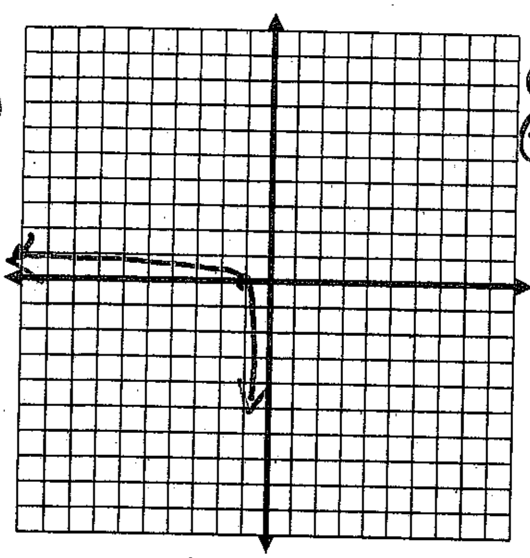
dilate $\times\frac{1}{2}$
up 2

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = 0$

2. Graph $g(x) = \log_{10}(-x)$



(-1,0)
(-10,1)

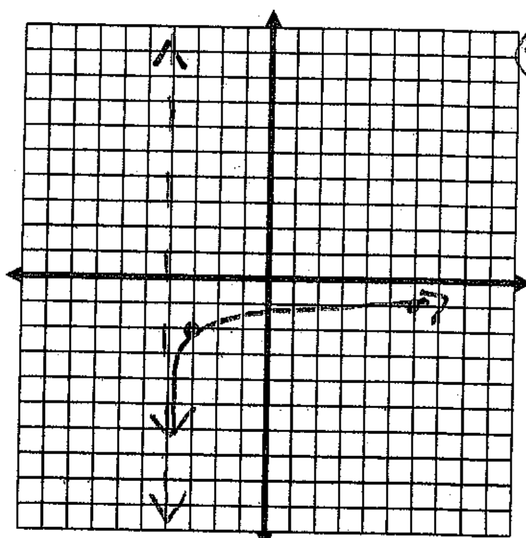
reflect y axis

Domain: $(-\infty, 0)$

Range: $(-\infty, \infty)$

Asymptote: $x = 0$

4. Graph $f(x) = \log_{10}(x+4) - 2$



(-3,-2)
(6,-1)

left 4
down 2

Domain: $(-4, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = -4$

Common Logs – have base 10 ... "$\log x$" means log base 10 of x

$\log_{10} 10 = \log_{10} 10^1 = 1$ $\quad$ $\log 1000 = \log_{10} 10^3 = 3$ $\quad$ $\log_{10} 40 \approx 1.6021 =$ ______

$\log_{10} 40 = x \quad 10^x = 40$

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