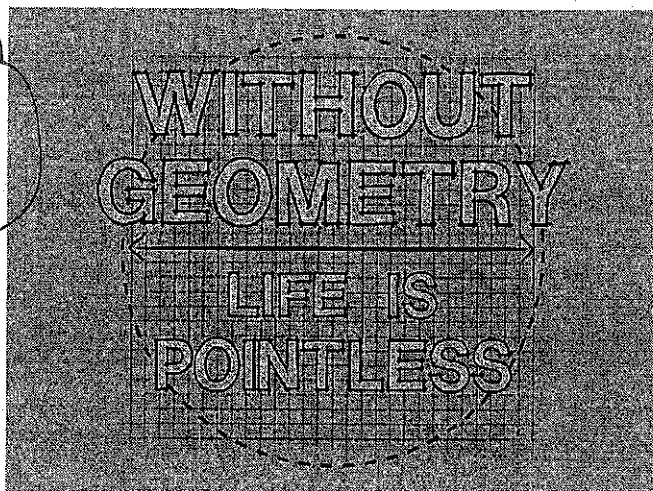


Math 3 – Fall 2017  
Unit 6 – Geometry and Circles

Day	Date	Lesson	Assignment
1	<del>12/11</del> 12/8	Tangent Lines of Circles Pages 1-4	
2	<del>12/13</del> 12/11	Radius, Chords, Angles and Arcs Pages 5-8	
3	<del>12/13</del> 12/12	Inscribed Angles, Chords and Arcs Pages 9-11	
4	<del>12/14</del> 12/13	Angle Measures and Segment Lengths Special Segments in a Circle Pages 12-14	
5	<del>12/15</del> 12/14	Equations of Circles Pages 15-18	
6	<del>12/18</del> 12/15	Area of Sectors and Arc Lengths Pages 19-21	
7	<del>12/19</del> 12/18	Review Pages 22-27	
8	<del>12/20</del> 12/19	Test	

12/20 STEM  
12/21 Project?



**I. Definitions:**

- A **circle** is defined as the set of all points in a plane that are equidistant from a given fixed point called the center. (360 degrees)
- A **radius** is a segment from the center to the edge of a circle.
- A **diameter** is the longest chord which contains the center.
- A **chord** is a segment whose endpoints are on the circle.
- A **central angle** is an angle whose vertex is located at the center of the circle and whose sides are radii.
- An **arc** is an unbroken part of the circle.
- A **minor arc** is an arc that is less than half the circle.  
measure = central angle measure
- A **major arc** is an arc that is more than half the circle.  
measure =  $360 - \text{central angle measure}$
- A **semicircle** is an arc made by the diameter of the circle. (180 degrees)
- **Adjacent arcs** have exactly one point in common.
- **Concentric circles** are circles that share the same center but have different radii.
- An **inscribed angle** has its vertex on the circle and its sides are chords of the circle.
- An **inscribe polygon** is a polygon where each vertex is on the circle.
- A **secant line** is a line that intersects a circle in two points.
- A **tangent line** is a line that intersects a circle in one point.
- The **point of tangency** is the point on the circle where the tangent intersects it.
- **Common external tangents** are tangents outside of two circles that do not intersect.
- **Common internal tangents** are tangents that intersect between two circles.
- A **circumscribed polygon** is a polygon where each side of the polygon is tangent to the circle.

**II. Theorems:**

- In a circle, if the diameter of the circle is perpendicular to a chord, then the diameter bisects the chord and its arc.
- Minor arcs are congruent if and only if their corresponding chords are congruent.
- In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.
- The measure of an inscribed angle is half the measure of the intercepted arc.
- If two inscribed angles of a circle or congruent circles intercept congruent arcs (or the same arc) then the angles are congruent.

- If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.
- In a plane, if a line is perpendicular to a radius of a circle at the endpoint on the circle, then the line is a tangent of the circle.
- If two segments from the same exterior point are tangent to a circle, then they are congruent.
- If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is  $\frac{1}{2}$  the measure of its intercepted arc.
- If 2 secants intersect in the interior of a circle, then the measure of an angle formed is  $\frac{1}{2}$  the sum of the measures of the arcs intercepted by the angle and its vertical angles.
- If 2 secants, a secant and a tangent, or 2 tangents intersect in the exterior of a circle, then the measure of the angle formed is  $\frac{1}{2}$  the positive differences of the measures of the intercepted arcs.
- If 2 chords intersect in a circle, then the products of the measures of the segments of the chords are equal.
- Two secants drawn to a circle from an exterior point, the product of the measures of one secant and its external secant segment is equal to the product of the measures of the other secant and its external secant segment.

III. Example Problems

1. Suppose a chord of a circle is 16cm long and is 12 cm from the center of the circle. Find the radius.



$$\begin{aligned} x^2 + 8^2 &= 12^2 \\ x^2 + 64 &= 144 \\ x^2 &= 80 \end{aligned}$$

$$\begin{aligned} x &= \sqrt{80} \\ &= \sqrt{16 \cdot 5} \\ &= 4\sqrt{5} \end{aligned}$$

Radius =  $\frac{\sqrt{80}}{4} = 4\sqrt{5} = 8.94$

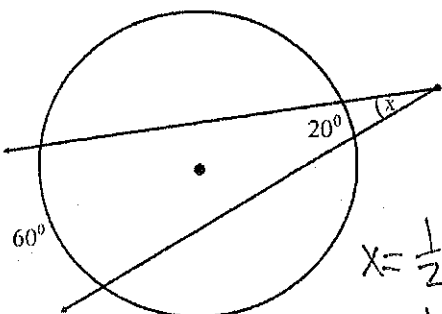
2. Suppose the diameter of a circle is 20 cm long, and a chord is 18 cm long. Find the distance between the chord and the center of the circle.



$$\begin{aligned} x^2 + 9^2 &= 10^2 \\ x^2 + 81 &= 100 \\ x^2 &= 19 \\ x &= \sqrt{19} \end{aligned}$$

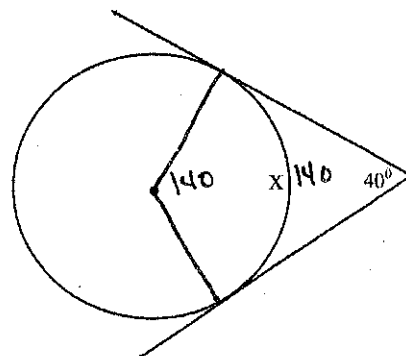
Distance =  $\frac{\sqrt{19}}{2} = 4.36$

3. Solve for x.  $x = 20^\circ$



$$\begin{aligned} x &= \frac{1}{2}(60 - 20) \\ &= \frac{1}{2}(40) \\ &= 20 \end{aligned}$$

4. Solve for x.  $x = 140$



Review--Unit 8: Circles  
Definitions and Theorems

exterior angle

Name \_\_\_\_\_

5. Solve for x.

x = 90

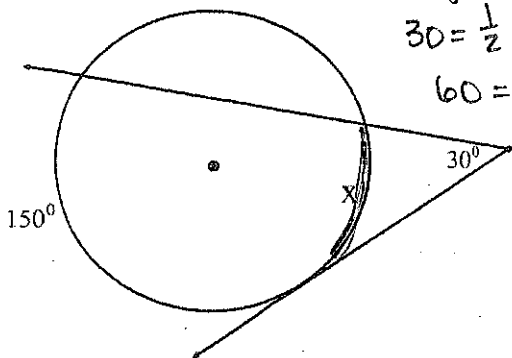
Angle =  $\frac{1}{2}$  Difference

$30 = \frac{1}{2} (150 - x)$

$60 = 150 - x$

$-90 = -x$

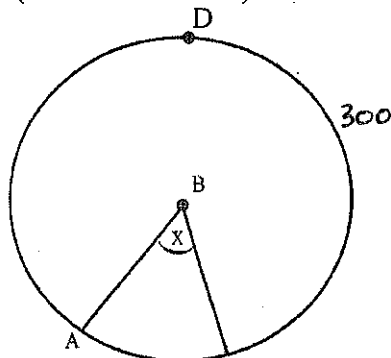
$x = 90$



6. Solve for x.

x = 60

(m arc ADC = 300°)



Central angle  
Angle = Arc

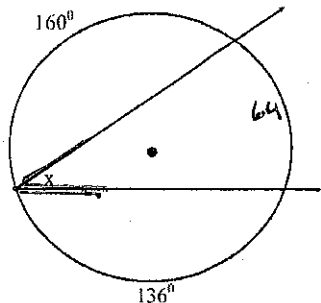
7. Solve for x.

x = 32

inscribed angle

Angle =  $\frac{1}{2}$  arc

$x = \frac{1}{2} (64)$



8. Solve for x.

x = 118

interior angle

Angle =  $\frac{1}{2}$  sum

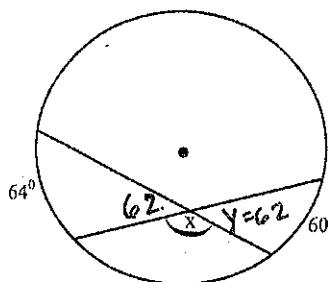
$y = \frac{1}{2} (64 + 60)$

$\frac{1}{2} (124)$

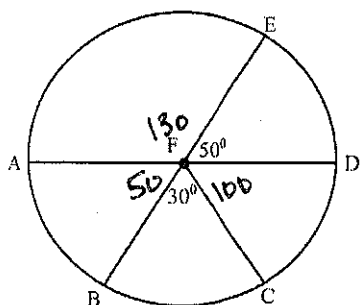
62

$x = 180 - 62$

$x = 118$



9.

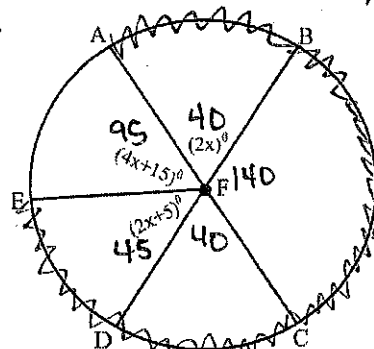


central angles

m arc DEC = 260 =  $360 - 100$

m arc AE = 130

10.



m arc CB = 140

m arc ACE = 265

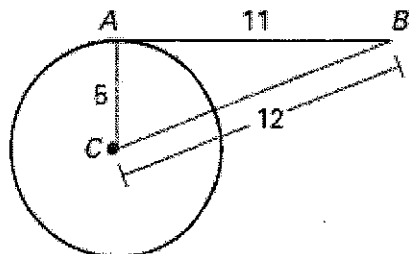
$2x + 5 + 4x + 15 + 2x = 180$

$8x + 20 = 180$

$8x = 160$

$x = 20$

11. Determine whether  $\overline{AB}$  is tangent to  $\odot C$ . Explain your reasoning.



Check

$5^2 + 11^2 \stackrel{?}{=} 12^2$

$25 + 121 = 144$

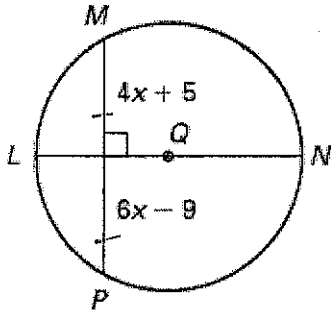
$146 \neq 144$

$\overline{AB}$  is not tangent, since not a right triangle. (26)

Review--Unit 8: Circles  
Definitions and Theorems

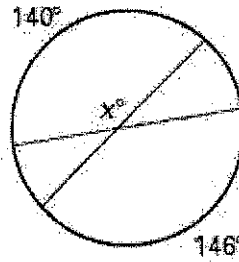
Name \_\_\_\_\_

12. Solve for x.  $x = \underline{7}$



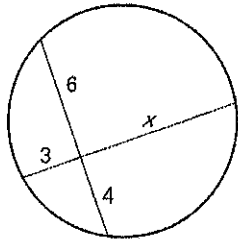
$$\begin{aligned} 4x + 5 &= 6x - 9 \\ 14 &= 2x \\ 7 &= x \end{aligned}$$

13. Solve for x.  $x = \underline{143}$



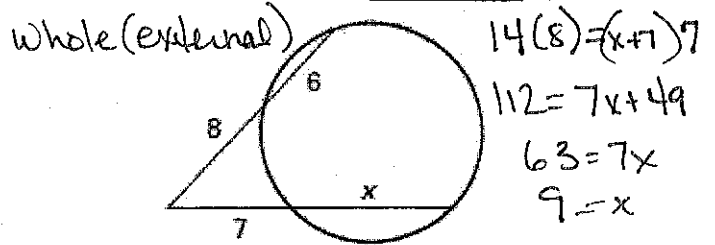
$$\begin{aligned} x &= \frac{1}{2}(140 + 146) \\ &= \frac{1}{2}(286) \\ &= 143 \end{aligned}$$

14.  $6 \cdot \underline{4} = 3 \cdot \underline{x}$



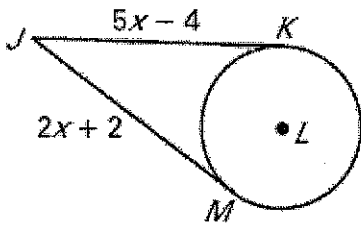
$$3x = 24$$

15. Solve for x.  $x = \underline{\hspace{2cm}}$

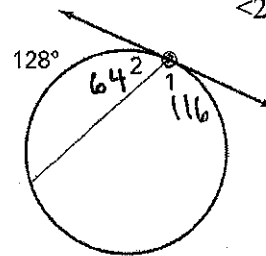


$$\begin{aligned} 14(8) &= (x+7) \cdot 7 \\ 112 &= 7x + 49 \\ 63 &= 7x \\ 9 &= x \end{aligned}$$

16. Solve for x.  $x = \underline{8}$



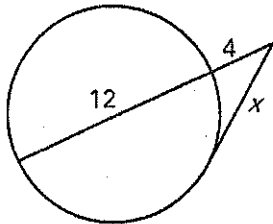
17. Find angles 1 and 2.  $\angle 1 = \underline{64}$



$\angle 2 = \underline{116}$

inscribed angle

18.  $x^2 = 4 \cdot \underline{16}$



whole (external)

$$\begin{aligned} 16(4) &= x^2 \\ 64 &= x^2 \\ 8 &= x \end{aligned}$$

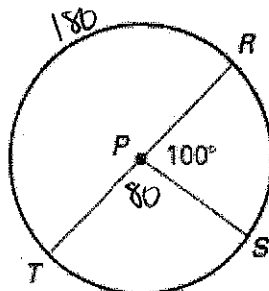
Find the measure of each arc of  $\odot P$ , where  $\overline{RT}$  is a diameter.

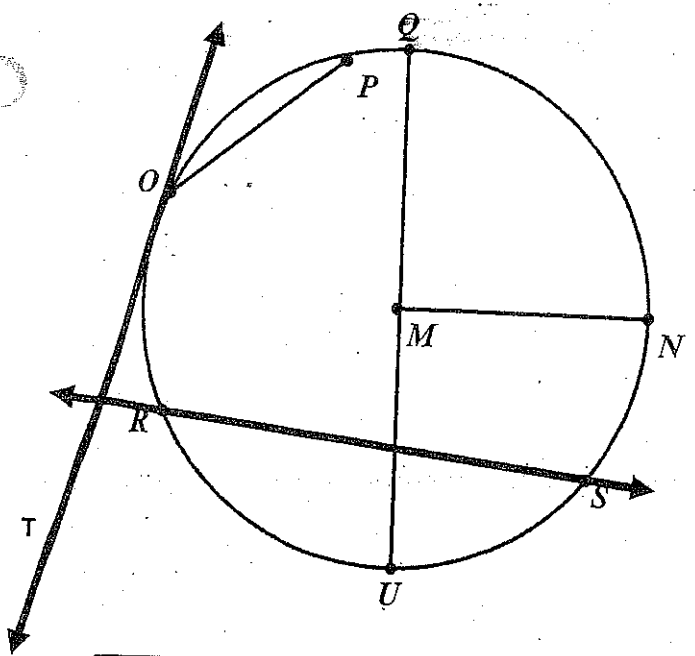
19.  $m \widehat{RS} = \underline{100}$

20.  $m \widehat{ST} = \underline{80}$

21.  $m \widehat{RTS} = \underline{200} = 180 + 80$

22.  $m \widehat{RST} = \underline{180}$





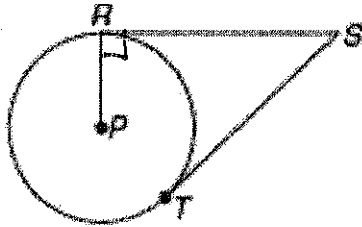
Term	Definition	Name
Circle	*	
Chord		
Radius		
Diameter		
Secant		
Tangent		
Arc		
Minor arc	Use 2 letters to name	
Major Arc	Use 3 letters to name	
Semicircle		

# 6.7 Tangent Lines of Circles

SWBAT solve for unknown variables using theorems about tangent lines of circles.

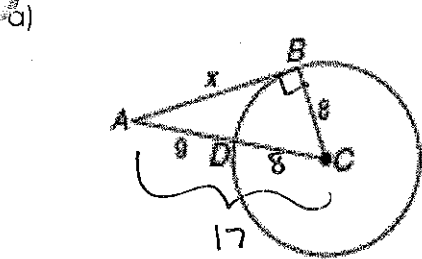
<p><b>Tangent to a Circle</b> Ex: <math>\overline{AB}</math></p>	<p>A line in the plane of the circle that intersects the circle in exactly one point. <b>Ex:</b> Segment <math>\overline{AB}</math> is a tangent to Circle <math>O</math>.</p>	
<p><b>Point of Tangency</b></p>	<p>The point where a circle and a tangent intersect. <b>Ex:</b> Point <math>P</math> is a point of tangency on Circle <math>O</math>.</p>	

<p><b>Tangent Theorem 1:</b> If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.</p>	<p><b>Converse Theorem 1:</b> If a line is perpendicular to the radius of a circle at its endpoint on a circle, then the line is tangent to the circle.</p>
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**Example:** If  $\overline{RS}$  is tangent, then  $\overline{PR} \perp \overline{RS}$ .

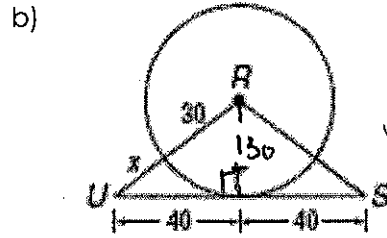
**Example 1:** Find the measure of  $x$ .



$$x^2 + 8^2 = 17^2$$

$$x^2 = 225$$

$$x = 15$$



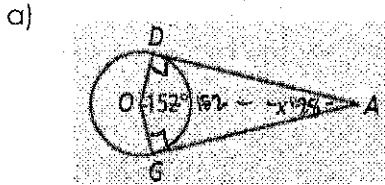
$$40^2 + 30^2 = (x + 30)^2$$

$$\sqrt{2500} = \sqrt{(x + 30)^2}$$

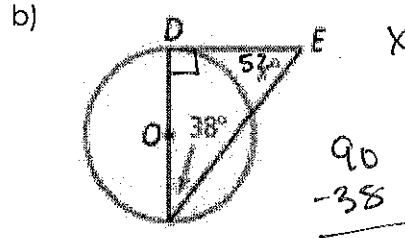
$$50 = x + 30$$

$$20 = x$$

**Example 2:** Find  $x$ . All segments that appear tangent are tangent to Circle  $O$ .

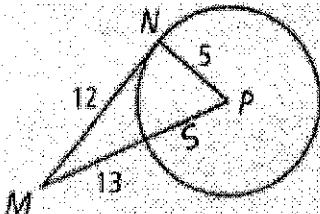


$$x = 28$$



$$x = 52$$

**Example 3:** Is segment  $\overline{MN}$  tangent to Circle  $O$  at  $P$ ? Explain.



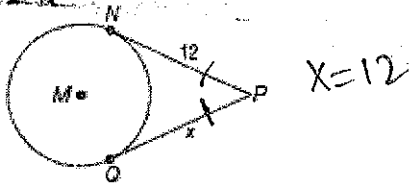
$$5^2 + 12^2 \stackrel{?}{=} 13^2$$

$$169 \neq 324$$

**Tangent Theorem 2:**

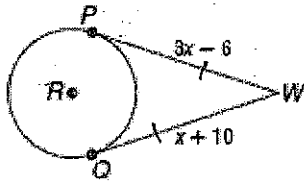
If two tangent segments to a circle share a common endpoint outside the circle, then the two segments are congruent.

**Example 4:** Solve for x.



$x = 12$

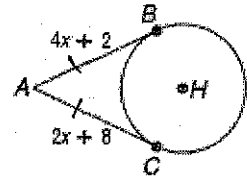
Ice cream cones



$$3x - 6 = x + 10$$

$$2x = 16$$

$$x = 8$$



$$4x + 2 = 2x + 8$$

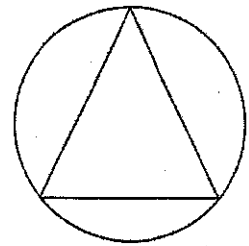
$$2x = 6$$

$$x = 3$$

**Circumscribed vs. Inscribed**

To **circumscribe** is when you draw a figure around another, touching it at points as possible.

To **inscribe** is to draw a figure within another so that the inner figure lies entirely within the boundary of the outer.

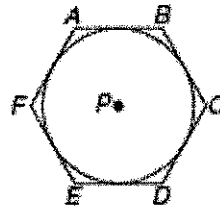
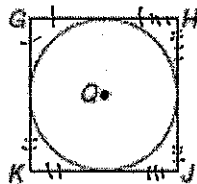


**Ex:** The circle is circumscribed about the triangle.

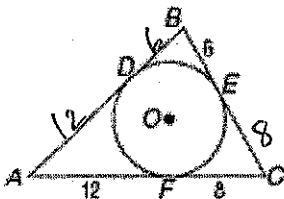
**Ex:** The triangle is inscribed in the circle.

**Tangent Theorem 3: (Circumscribed Polygons)**

When a polygon is circumscribed about a circle, all of the sides of the polygon are tangent to the circle.



**Example 5:** Triangle ABC is circumscribed about  $\odot O$ . Find the perimeter of triangle ABC.

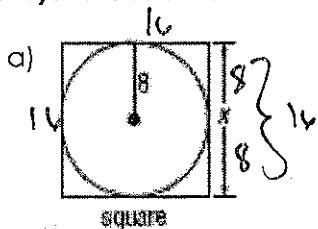


$$P = 2(12) + 2(6) + 2(8)$$

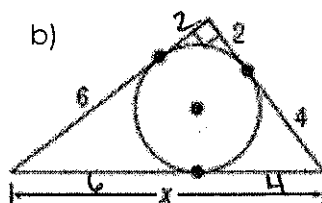
$$P = 24 + 12 + 16$$

$$P = 52$$

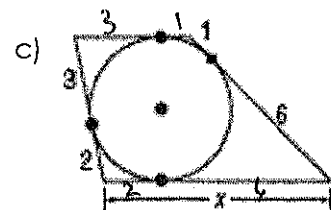
**You Try!** Find x. Assume that segments that appear to be tangent are tangent.



$x = 14$



$x = 10$



$x = 8$

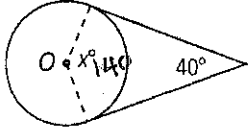


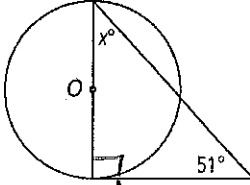
# Practice 6.7: Tangents of Circles

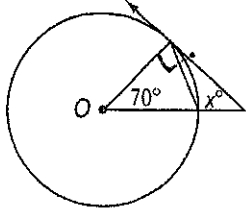
Name: \_\_\_\_\_

Math 3

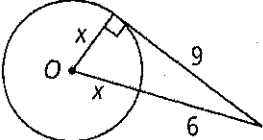
**Directions:** Assume that lines that appear to be tangent are tangent. O is the center of each circle. What is the value of x?

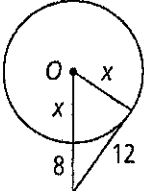
1.   
 $x = 180 - 40 = 140$

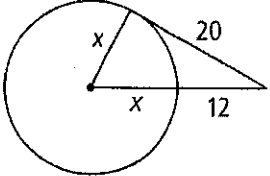
2.   
 $x = 90 - 51 = 39$

3.   
 $x = 90 - 70 = 20$

**Directions:** In each circle, what is the value of x to the nearest tenth?

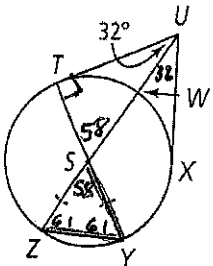
4.   
 $x^2 + 9^2 = (x+6)^2$   
 $x^2 + 81 = x^2 + 12x + 36$   
 $\frac{45}{12} = \frac{12x}{12} \quad x \approx 3.8$

5.   
 $x^2 + 12^2 = (x+8)^2$   
 $x^2 + 144 = x^2 + 16x + 64$   
 $80 = 16x$   
 $S = X$

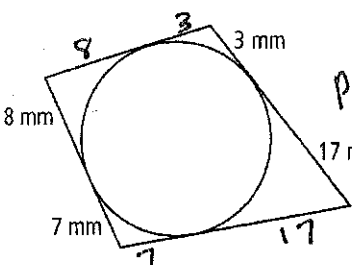
6.   
 $x^2 + 20^2 = (x+12)^2$   
 $x^2 + 400 = x^2 + 24x + 144$   
 $256 = 24x$   
 $x \approx 10.7$

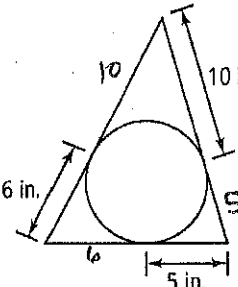
7.  $\overline{TY}$  and  $\overline{ZW}$  are diameters of  $\odot S$ .  $\overline{TU}$  and  $\overline{UX}$  are tangents of  $\odot S$ . What is  $m\angle SYZ$ ?

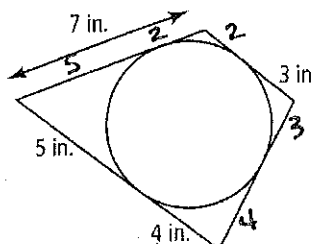
$$\begin{array}{r} 180 \\ - 58 \\ \hline 2 \overline{) 122} \underline{61} \end{array}$$

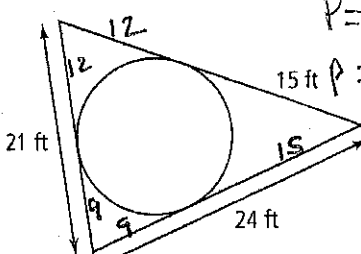


**Directions:** Each polygon circumscribes a circle. What is the perimeter of each polygon?

8.   
 $P = 2(8 + 3 + 17 + 7)$   
 $P = 2(35)$   
 $P = 70$

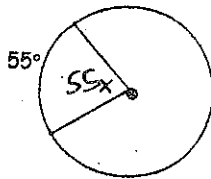
9.   
 $P = 2(10 + 5 + 6)$   
 $P = 2(21)$   
 $P = 42$

10.   
 $P = 2(2 + 3 + 4 + 5)$   
 $P = 2(14)$   
 $P = 28$

11.   
 $P = 2(12 + 15 + 9)$   
 $P = 2(36)$   
 $P = 72$

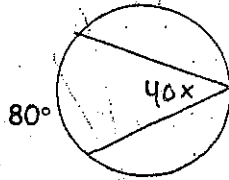
1) Central Angle = to its intercepted arc

$x = \underline{55}$       Angle = Arc



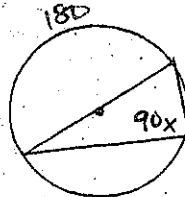
2) Inscribed Angle =  $\frac{1}{2}$  its intercepted arc

$x = \underline{40}$       Angle =  $\frac{1}{2}$  Arc



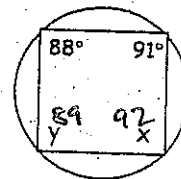
3) An inscribed angle in a semicircle =  $90^\circ$

$x = \underline{90}$



4) Opposite angles of an inscribed quadrilateral are supplementary

$x = \underline{92}$      $y = \underline{89}$



5) Angle formed by two chords intersecting in the interior of a circle =  $\frac{\text{arc} + \text{arc}}{2}$

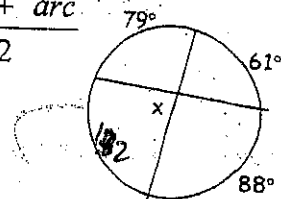
$x = \underline{96.5}$

Angle =  $\frac{1}{2}$  sum

$x = \frac{1}{2}(182 + 61)$

~~Angle =  $\frac{1}{2}$  sum~~

$x = \frac{1}{2}(193) = 96.5$



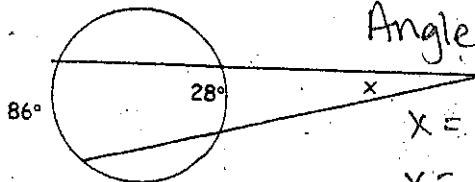
6) Angle formed by two secant segments in the exterior of the circle =  $\frac{\text{outer arc} - \text{inner arc}}{2}$

$x = \underline{29}$

Angle =  $\frac{1}{2}$  Difference

$x = \frac{1}{2}(86 - 28)$

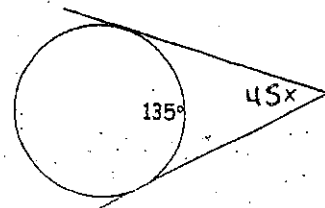
$x = \frac{1}{2}(58) = 29$



{Cone Head}

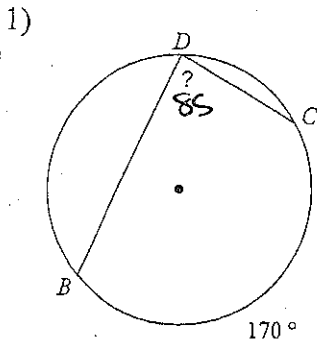
7) Angle formed by two tangents intersecting in the exterior of the circle =  $180 - \text{inner arc}$

$x = \underline{45}$

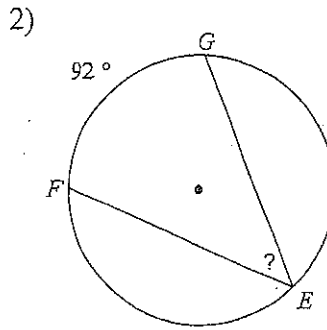


# Arcs and Angles of Circles

Find the measure of the arc or angle indicated.

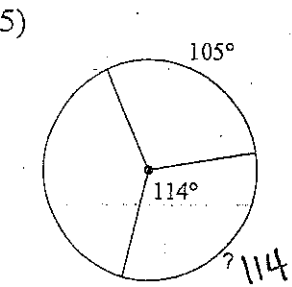
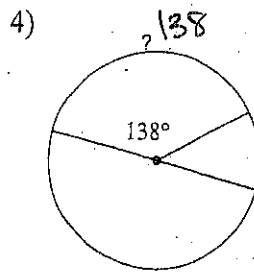
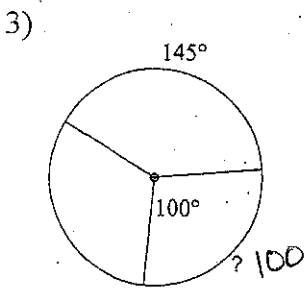


$$\frac{1}{2}(170) = 85$$

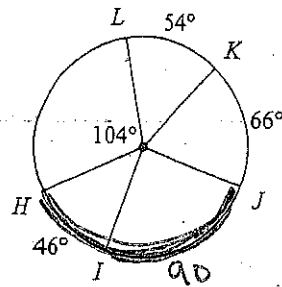


$$\text{Angle} = \frac{1}{2} \text{ Inscribed} = \frac{1}{2}(92) = 46$$

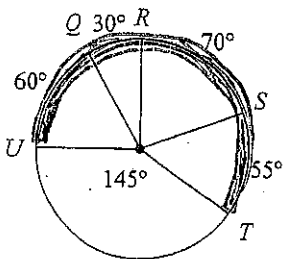
Find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.



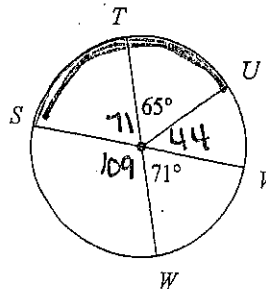
6)  $m\widehat{JH} = 90 + 46 = 136$



7)  $m\widehat{URT} = 215$



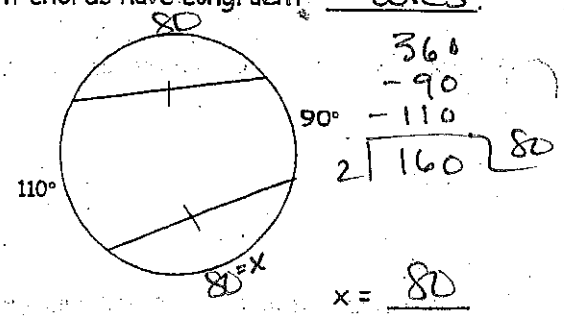
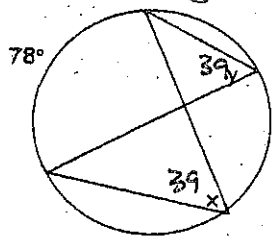
8)  $m\widehat{SU} = 71 + 65 = 136$



8) Angles inscribed in the same arc or intercepting congruent arcs are congruent

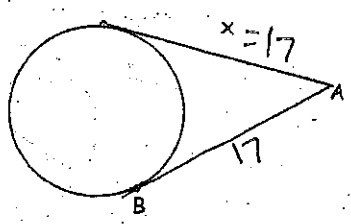
9) Congruent chords have congruent arcs

$x = 39$   $y = 39$



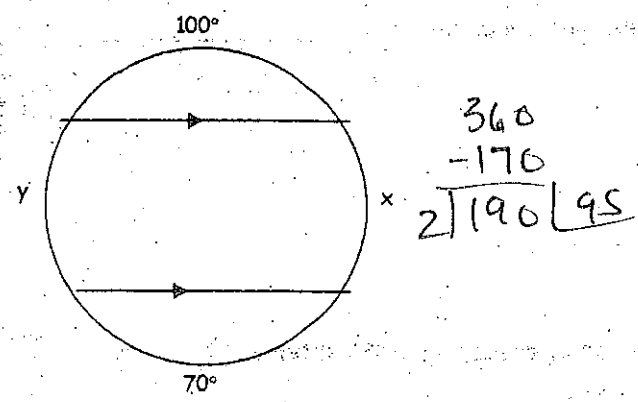
(Cone Head) Ice Cream Cone.  
Two tangent segments from an external point to a circle are congruent

If  $AB = 17$ ,  $x = 17$



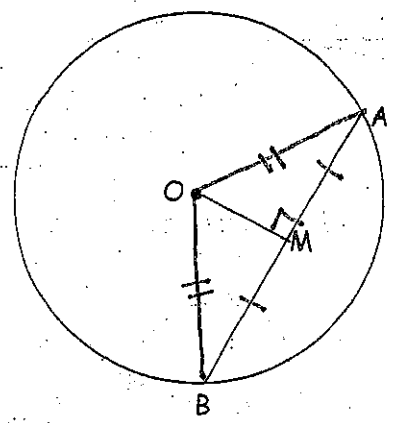
10) In a circle, parallel chords intercept congruent arcs

$x = 95$   
 $y = 95$

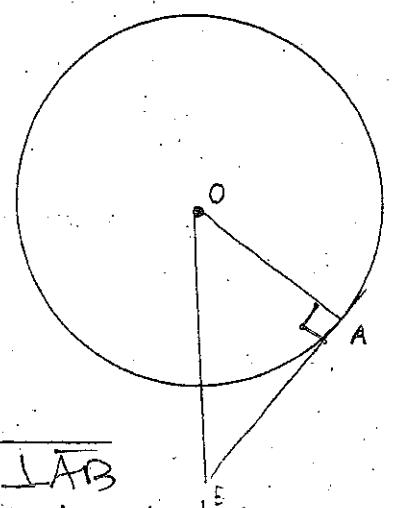


Triple Theorems: if any 2 of the following are true, the third statement is also true.

- 11) a) line through the center of a circle
- b) line perpendicular to a chord
- c) line bisecting the chord
- 12) a) line contains the center of a circle
- b) line tangent to the circle
- c) line perpendicular to a radius at point of tangency



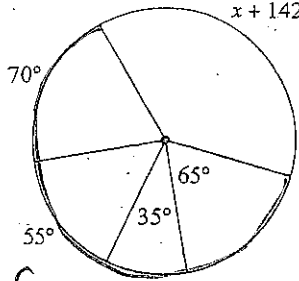
Conclusion:  
 $\overline{OM} \perp \overline{AB}$   
 $\overline{AM} \cong \overline{MB}$   
 $\triangle BOM \cong \triangle AOM$



Conclusion:  
 $\overline{OA} \perp \overline{EA}$   
 $\triangle AEO$  is a right triangle

6

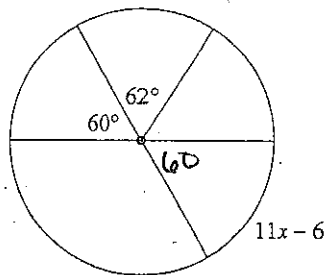
9)  $x + 142 = 135$



$x + 142 = 135$   
 $x = -7$

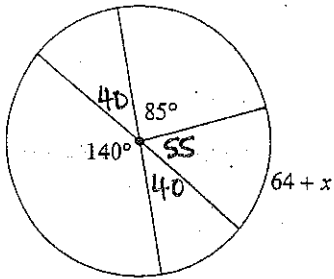
225

10)



$11x - 6 = 60$   
 $11x = 66$   
 $x = 6$

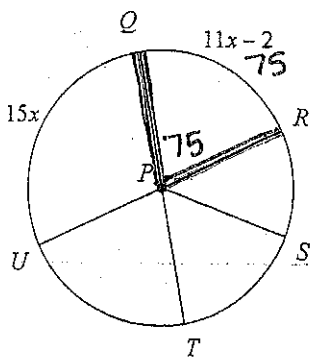
11)



$x + 64 = 55$   
 $x = -9$

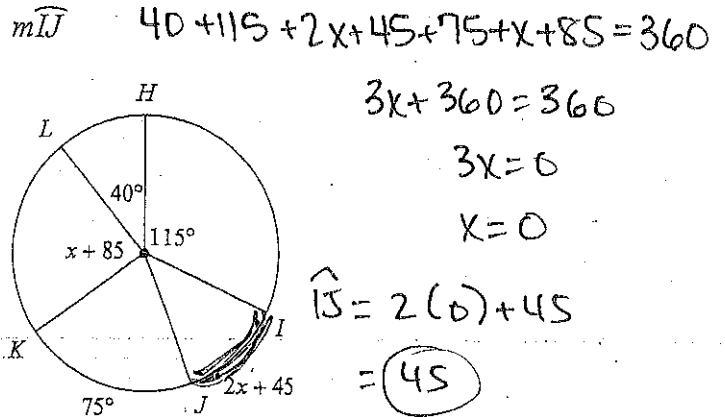
Find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.

12)  $m\angle QPR$



$15x + 11x - 2 = 180$   
 $26x = 182$   
 $x = 7$   
 $\hat{QR} = 11x - 2$   
 $11(7) - 2$   
 $77 - 2$   
 $75$

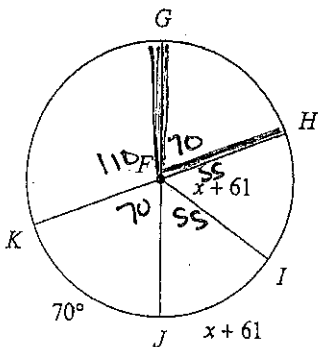
13)  $m\widehat{IJ}$



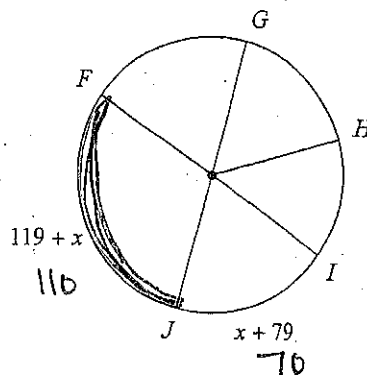
$40 + 115 + 2x + 45 + 75 + x + 85 = 360$   
 $3x + 360 = 360$   
 $3x = 0$   
 $x = 0$   
 $\hat{I} = 2(0) + 45$   
 $= 45$

14)  $m\angle GFH$

70



15)  $m\widehat{JF} = 110$

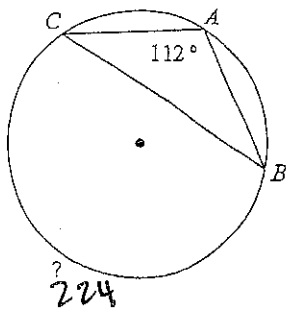


$119 + x + x + 79 = 180$   
 $2x + 198 = 180$   
 $2x = -18$   
 $x = -9$

8

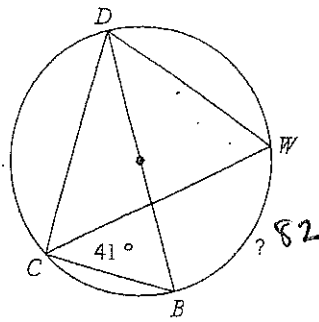
Find the measure of the arc or angle indicated.

1)

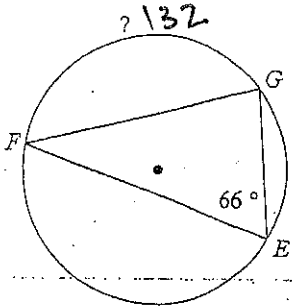


$Angle = \frac{1}{2} Arc$

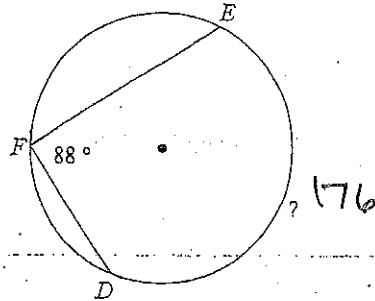
2)



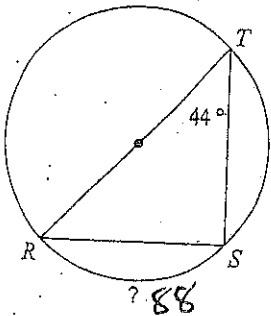
3)



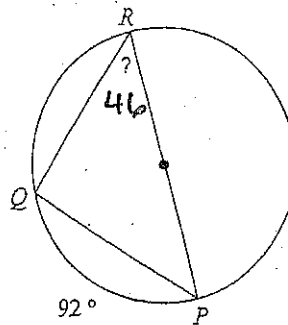
4)



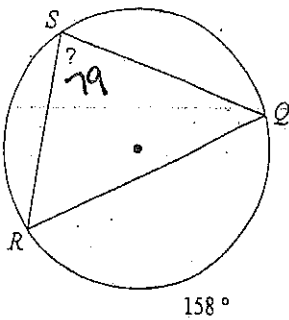
5)



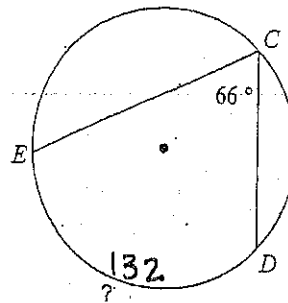
6)



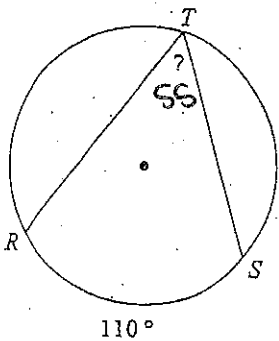
7)



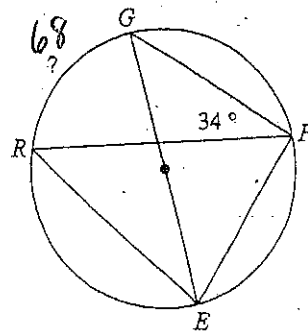
8)



9)



10)



$2(8x+8) = 17x+7$   
 $16x+16 = 17x+7$   
 $9 = x$

$2x+105+107+148=360$   
 $2x+360=360$   
 $2x=0$   
 $x=0$

23)

$9x+3=120$

$9x+3=120$   
 $9x=117$   
 $x=13$

24)

$50+75+12x-S=180$   
 $12x+120=180$   
 $12x=60$   
 $x=5$

25)

$8x+7=55$   
 $8x=48$   
 $x=6$

$8x+7=55$   
 $8x=48$   
 $x=6$

26)

$11x+2$   
 $22x-2$   
 $+ 12x$   
 $45x=360$   
 $x=8$

$11x+2$   
 $22x-2$   
 $+ 12x$   
 $45x=360$   
 $x=8$

27)

$11x+5+120=180$   
 $11x+125=180$   
 $11x=55$   
 $x=5$

$11x+5+120=180$   
 $11x+125=180$   
 $11x=55$   
 $x=5$

28)

$2(4x+17)=10x+12$   
 $8x+34=10x+12$   
 $22=2x$   
 $11=x$

$2(4x+17)=10x+12$   
 $8x+34=10x+12$   
 $22=2x$   
 $11=x$

29)

$19x+9$   
 $17x+7$   
 $14x-6$   
 $50x+10=360$   
 $50x=350$   
 $x=7$

$19x+9$   
 $17x+7$   
 $14x-6$   
 $50x+10=360$   
 $50x=350$   
 $x=7$

30)

$110$   
 $111x-1=110$   
 $111x=111$   
 $x=1$

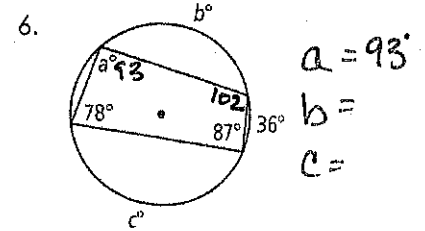
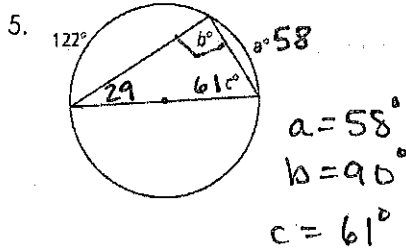
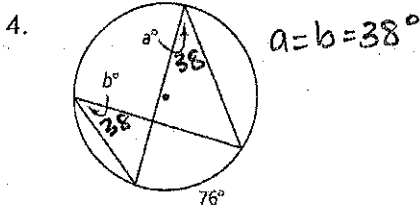
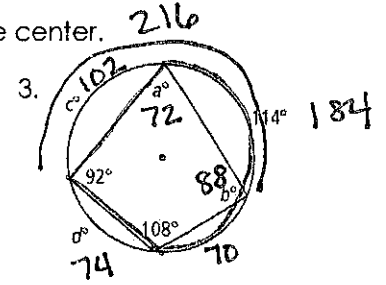
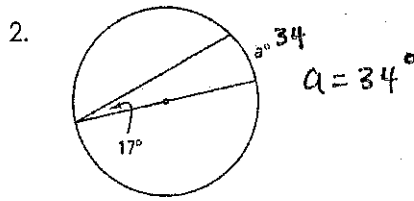
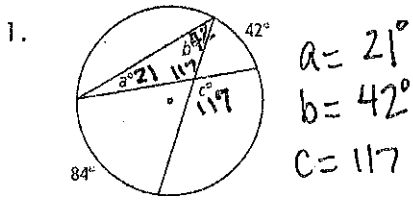
$111x-1=110$   
 $111x=111$   
 $x=1$

# Homework 6.9: Inscribed Angles

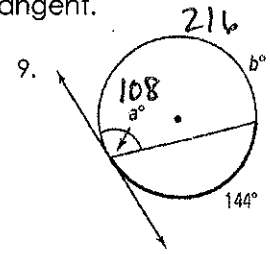
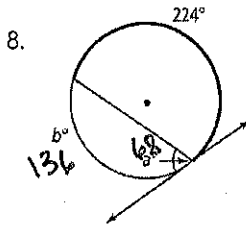
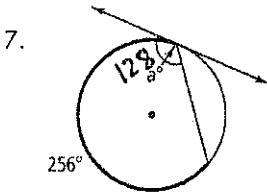
Name: \_\_\_\_\_

Math 3

**Directions:** Find the value of each variable. For each circle, the dot represents the center.



**Directions:** Find the value of each variable. Lines that appear to be tangent are tangent.



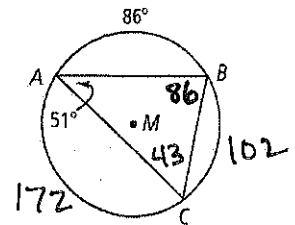
**Directions:** Find each indicated measure for  $\odot M$ .

10.  $m\angle B = 86^\circ$

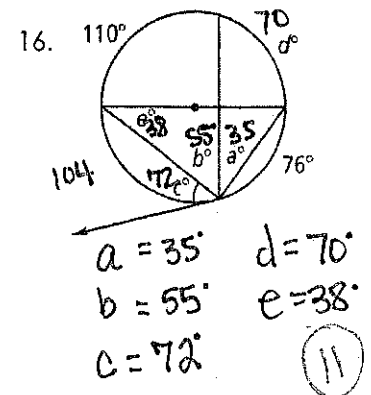
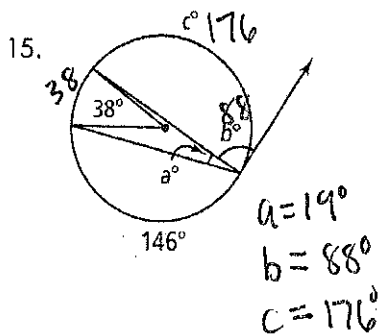
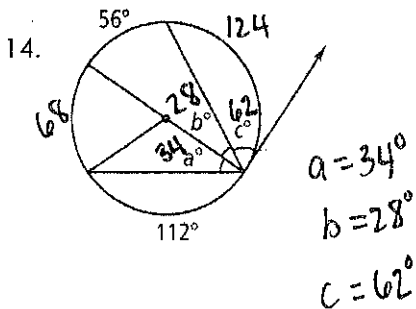
11.  $m\angle C = 43^\circ$

12.  $m\widehat{BC} = 102^\circ$

13.  $m\widehat{AC} = 172^\circ$



**Directions:** Find the value of each variable. For each circle, the dot represents the center.

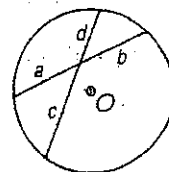




# Study Guide and Intervention

## Special Segments in a Circle

**Segments Intersecting Inside a Circle** If two chords intersect in a circle, then the products of the measures of the chords are equal.



$$a \cdot b = c \cdot d$$

**Example**

Find  $x$ .

The two chords intersect inside the circle, so the products  $AB \cdot BC$  and  $EB \cdot BD$  are equal.

$$AB \cdot BC = EB \cdot BD$$

$$6 \cdot x = 8 \cdot 3$$

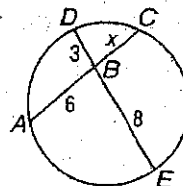
$$6x = 24$$

$$x = 4$$

Substitution

Simplify.

Divide each side by 6.

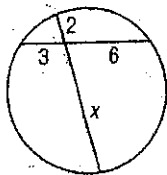


$$AB \cdot BC = EB \cdot BD$$

**Exercises**

Find  $x$  to the nearest tenth.

1.

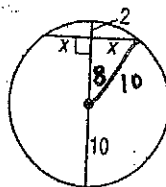


$$2x = 3(6)$$

$$2x = 18$$

$$x = 9$$

2.

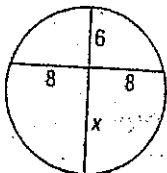


$$x^2 + 8^2 = 10^2$$

$$x^2 = 36$$

$$x = 6$$

3.

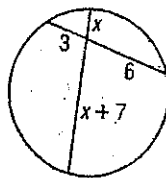


$$6x = 64$$

$$x = \frac{64}{6} = \frac{32}{3} = 10\frac{2}{3}$$

$$x \approx 10.7$$

4.



$$x(x+7) = 3(6)$$

$$x^2 + 7x = 18$$

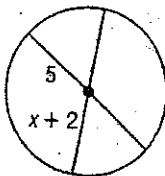
$$x^2 + 7x - 18 = 0$$

$$(x+9)(x-2) = 0$$

$$x = -9$$

$$x = 2$$

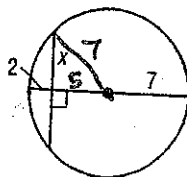
5.



$$x+2 = 5$$

$$x = 3$$

6.

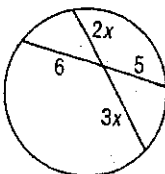


$$x^2 + 5^2 = 7^2$$

$$\sqrt{x^2} = \sqrt{124}$$

$$x = 4.9$$

7.



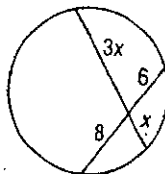
$$2x(3x) = 30$$

$$6x^2 = 30$$

$$x^2 = 5$$

$$x = \sqrt{5} \approx 2.2$$

8.



$$3x(x) = 48$$

$$3x^2 = 48$$

$$x^2 = 16$$

$$x = 4$$

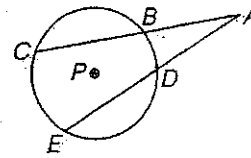
$$(12)$$

9-7 Study Guide and Intervention (continued)

Special Segments in a Circle

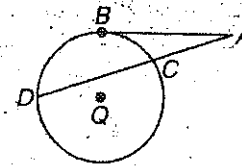
**Segments Intersecting Outside a Circle** If secants and tangents intersect outside a circle, then two products are equal.

- If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.



$\overline{AC}$  and  $\overline{AE}$  are secant segments.  
 $\overline{AB}$  and  $\overline{AD}$  are external secant segments.  
 $AC \cdot AB = AE \cdot AD$

- If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.



$\overline{AB}$  is a tangent segment.  
 $\overline{AD}$  is a secant segment.  
 $\overline{AC}$  is an external secant segment.  
 $(AB)^2 = AD \cdot AC$

**Example** Find  $x$  to the nearest tenth.

The tangent segment is  $\overline{AB}$ , the secant segment is  $\overline{BD}$ , and the external secant segment is  $\overline{BC}$ .

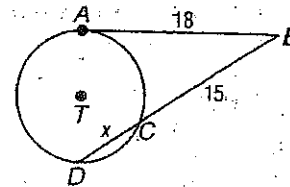
$$(AB)^2 = BC \cdot BD$$

$$(18)^2 = 15(15 + x)$$

$$324 = 225 + 15x$$

$$99 = 15x$$

$$6.6 = x$$



whole (external)

Exercises

Find  $x$  to the nearest tenth. Assume segments that appear to be tangent are tangent.

1.  $2.2(x + 2.2) = (3.3)(3.3)$   
 $2.2x + 4.84 = 10.89$   
 $2.2x = 6.05$   
 $x = 2.75$

2.  $42(16) = (x + 18)18$   
 $672 = 18x + 324$   
 $348 = 18x$   
 $19.3 = x$

3.  $16(8) = (2x + 6)6$   
 $128 = 12x + 36$   
 $92 = 12x$   
 $7.7 = x$

4.  $13(13) = (x + 9)9$   
 $169 = 45x + 81$   
 $88 = 45x$   
 $2.0 = x$

5.  $(4x + 6)6 = (x + 3)6$   
 $24x + 36 = 6x + 54$   
 $18x = 18$   
 $x = 1$

6.  $10(8) = (11 + x)x$   
 $80 = 11x + x^2$   
 $x^2 + 11x - 80 = 0$   
 $(x + 16)(x - 5) = 0$   
 $x = 16$  or  $x = 5$

7.  $35(35) = (x + 21)21$   
 $1225 = 21x + 441$   
 $784 = 21x$   
 $37.3 = x$

8.  $20(5) = x \cdot x$   
 $100 = x^2$   
 $10 = x$

9.  $8.8 = (x + 6)x$   
 $64 = x^2 + 6x$   
 $x^2 + 6x - 64 = 0$   
 $x = \frac{-6 \pm \sqrt{36 - 4(1)(-64)}}{2}$   
 $= \frac{-6 \pm \sqrt{292}}{2}$   
 $= 5.5$  or  $-11.5$

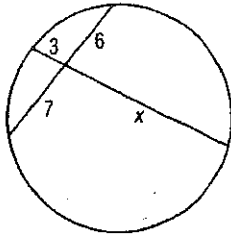
P1

# Skills Practice

## Special Segments in a Circle

Find  $x$  to the nearest tenth. Assume that segments that appear to be tangent are tangent.

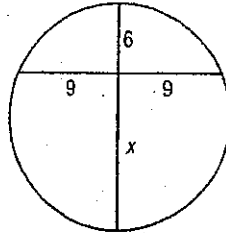
1.



$$3x = 42$$

$$x = 14$$

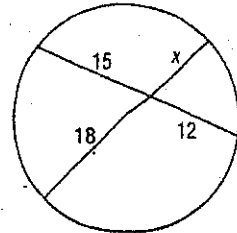
2.



$$6x = 81$$

$$x = 13.5$$

3.

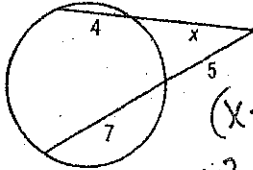


$$18x = 15(12)$$

$$18x = 180$$

$$x = 10$$

4.



$$(x+4)x = 12(5)$$

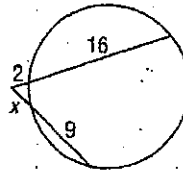
$$x^2 + 4x = 60$$

$$x^2 + 4x - 60 = 0$$

$$(x+10)(x-6) = 0$$

$$x = -10 \quad x = 6$$

5.



$$18(2) = (x+9)x$$

$$36 = x^2 + 9x$$

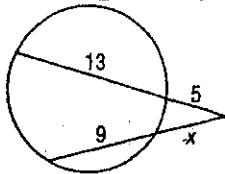
$$0 = x^2 + 9x - 36$$

$$0 = (x+12)(x-3)$$

$$x = -12$$

$$x = 3$$

6.



$$18(5) = (x+9)x$$

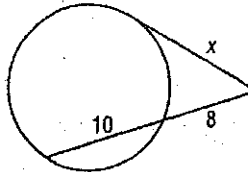
$$90 = x^2 + 9x$$

$$x^2 + 9x - 90 = 0$$

$$(x+15)(x-6) = 0$$

$$x = -15 \quad x = 6$$

7.

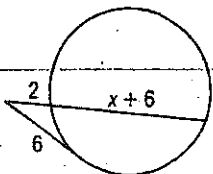


$$x^2 = 18(8)$$

$$x^2 = 144$$

$$x = 12$$

8.



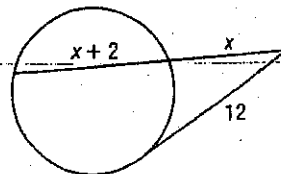
$$6(6) = (x+6)2$$

$$36 = 2x + 16$$

$$20 = 2x$$

$$10 = x$$

9.



$$(2x+2)x = 12(12)$$

$$2x^2 + 2x = 144$$

$$2x^2 + 2x - 144 = 0$$

$$2(x^2 + x - 72) = 0$$

$$(x+9)(x-8) = 0$$

$$x = -9$$

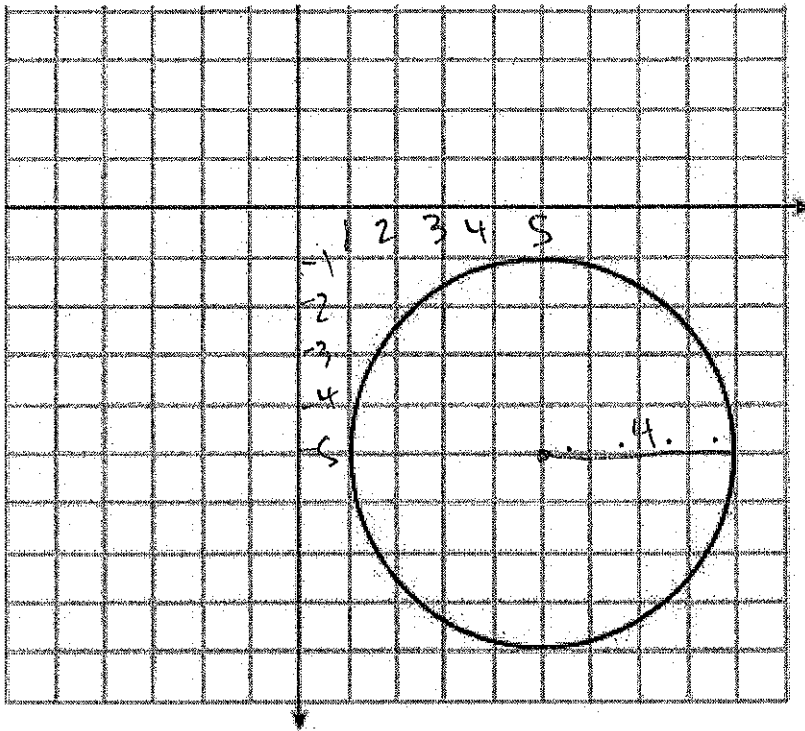
$$x = 8$$

14

# EQUATIONS OF CIRCLES

## Model Problem 2

What is the equation of the circle below?



Center =  $(5, -5)$   
Radius = 4

$$(x-5)^2 + (y+5)^2 = 16$$

## Model Problem 3 and 4

Identify the coordinates of the center and the length of the radius in the circles below

$(x-5)^2 + (y+2)^2 = 4$     radius: 2    Center:  $(5, -2)$

$(x+2)^2 + (y-1)^2 = 9$     radius: 3    Center:  $(-2, 1)$

## Part I

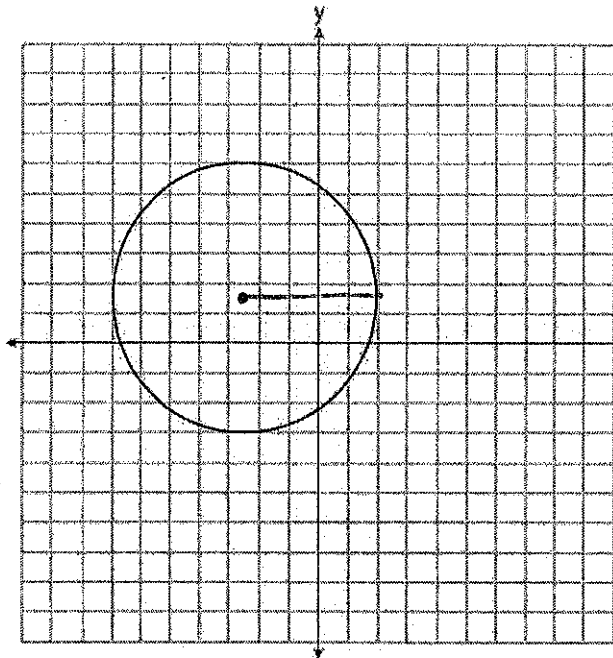
Identify the coordinates of the center and the length of the radius in the circles below.

1)  $(x-1)^2 + (y-3)^2 = 9$     radius: 3    Center:  $(1, 3)$

2)  $(x+14)^2 + (y-5)^2 = 16$     radius: 4    Center:  $(-14, 5)$

3)  $(x-5)^2 + (y-1)^2 = 25$     radius: 5    Center:  $(5, 1)$

4) What is the equation of the circle pictured below

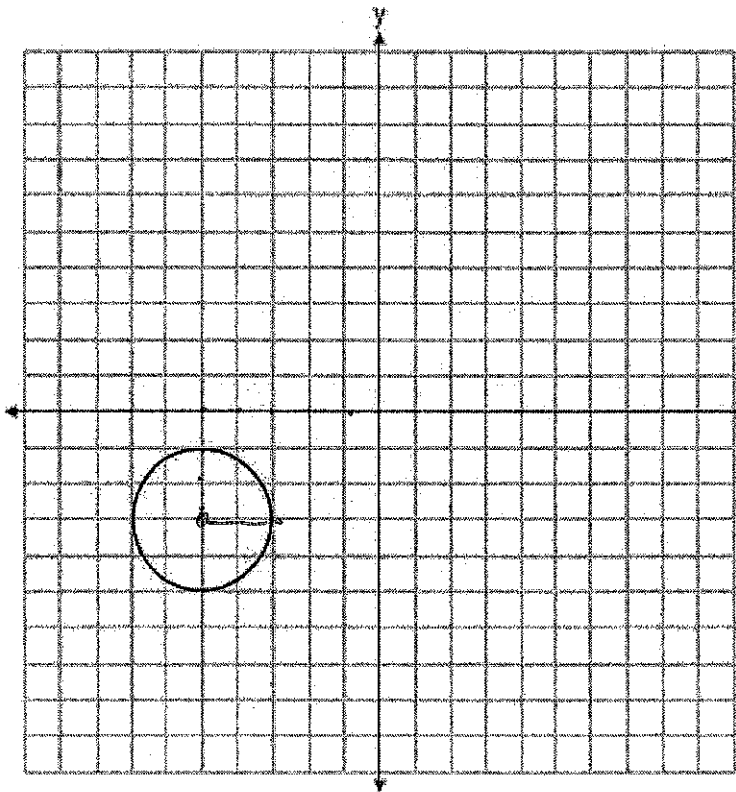


center:  $(-2.5, 1.5)$

radius: 4.5

$$(x + 2.5)^2 + (y - 1.5)^2 = 20.25$$

5) What is the equation of the circle pictured below



$r = 2$

center =  $(-5, -3)$

$$(x + 5)^2 + (y + 3)^2 = 4$$

# 9-8 Skills Practice

## Equations of Circles

Write an equation for each circle.

1. center at origin,  $r = 6$

$$x^2 + y^2 = 36$$

2. center at  $(0, 0)$ ,  $r = 2$

$$x^2 + y^2 = 4$$

3. center at  $(4, 3)$ ,  $r = 9$

$$(x-4)^2 + (y-3)^2 = 81$$

4. center at  $(7, 1)$ ,  $d = 24$   $r = 12$

$$(x-7)^2 + (y-1)^2 = 144$$

5. center at  $(-5, 2)$ ,  $r = 4$

$$(x+5)^2 + (y-2)^2 = 16$$

6. center at  $(6, -8)$ ,  $d = 10$   $r = 5$

$$(x-6)^2 + (y+8)^2 = 25$$

- ★ 7. a circle with center at  $(8, 4)$  and a radius with endpoint  $(0, 4)$

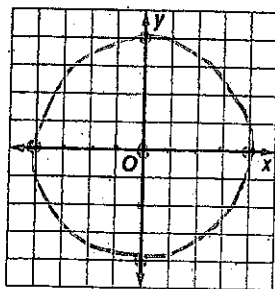
- ★ 8. a circle with center at  $(-2, -7)$  and a radius with endpoint  $(0, 7)$

- ★ 9. a circle with center at  $(-3, 9)$  and a radius with endpoint  $(1, 9)$

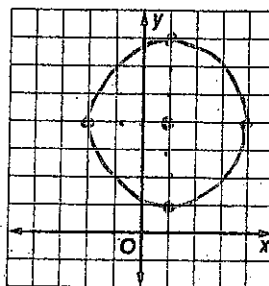
- ★ 10. a circle whose diameter has endpoints  $(-3, 0)$  and  $(3, 0)$

Graph each equation.

11.  $x^2 + y^2 = 16$



12.  $(x-1)^2 + (y-4)^2 = 9$



$(1, 4)$   
 $r = 3$

(17)

# Homework 6.10: Equations of Circles

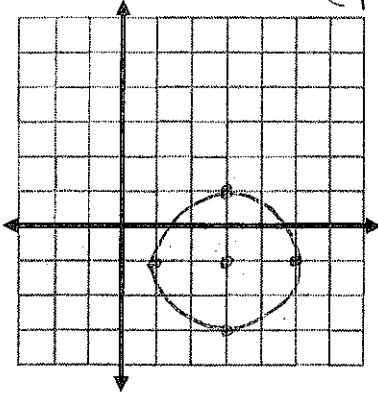
Name: \_\_\_\_\_

Math 3

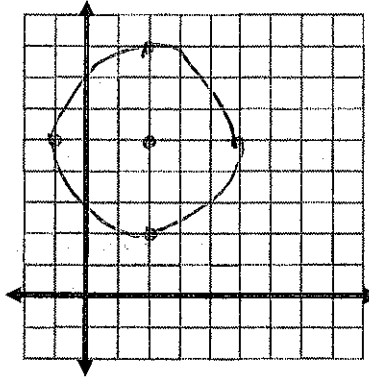
**Note:** If  $r^2$  is not a perfect square then leave  $r$  in simplified radical form but use the decimal equivalent for graphing. Example:  $\sqrt{12} = 2\sqrt{3} = 3.46$

1) Graph the following circle:

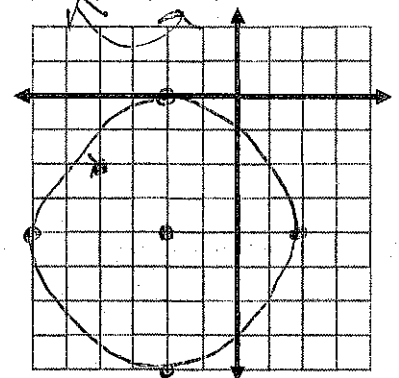
a.  $(x-3)^2 + (y+1)^2 = 4$   $(3, -1)$   
 $r=2$



b.  $(x-2)^2 + (y-5)^2 = 9$   $(2, 5)$   $r=3$



c.  $(y+4)^2 + (x+2)^2 = 16$   $(-2, -4)$   
 $r=4$



2) For each circle, identify its center and radius.

a.  $(x+3)^2 + (y-1)^2 = 4$

Center:  $(-3, 1)$

Radius:  $2$

b.  $x^2 + (y-3)^2 = 18$

Center:  $(0, 3)$

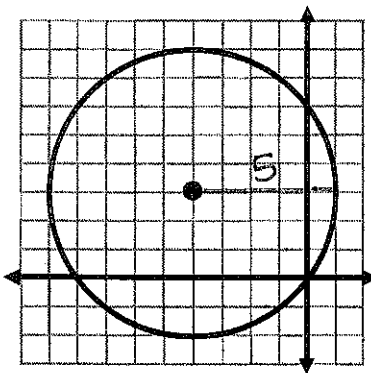
Radius:  $\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$   
 $= 4.24$

c.  $(y+8)^2 + (x+2)^2 = 72$

Center:  $(-2, -8)$

Radius:  $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$   
 $= 8.49$

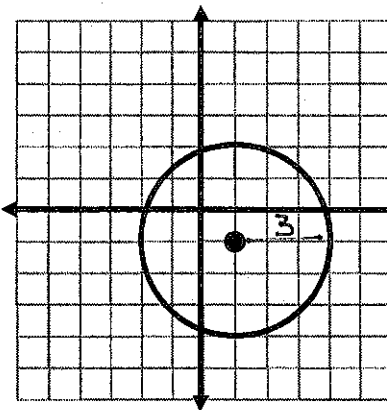
3) Write the equation of the following circles:



$(-4, 3)$

$r=5$

$(x+4)^2 + (y-3)^2 = 25$

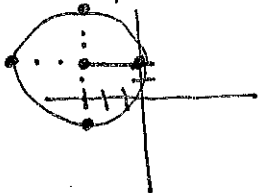


$(1, -1)$

$r=3$

$(x-1)^2 + (y+1)^2 = 9$

4) Give the equation of the circle that is tangent to the y-axis and center is  $(-3, 2)$ .  $r=3$



$(x+3)^2 + (y-2)^2 = 9$

5) Compare and contrast the following pairs of circles

a. Circle #1:  $(x-3)^2 + (y+1)^2 = 25$

Circle #2:  $(x+1)^2 + (y-2)^2 = 25$

Same radius  
 different centers

b. Circle #1:  $(y+4)^2 + (x+7)^2 = 6$

Circle #2:  $(x+7)^2 + (y+4)^2 = 36$

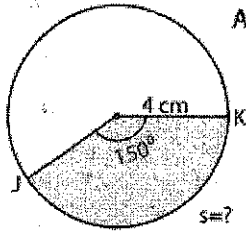
Same centers  
 different radius

# Practice: Arc Length & Area of Sectors

Name: \_\_\_\_\_

Math 3

Example:

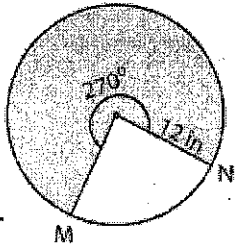


$$\begin{aligned} \text{Arc length of a sector (s)} &= \frac{\theta \times \pi \times r}{180^\circ} \\ &= \frac{150^\circ \times 3.14 \times 4}{180^\circ} \\ &= 10.47 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{\theta \pi r^2}{360} \\ &= \frac{150 \cdot 3.14 \cdot 4^2}{360} \\ &= 20.94 \text{ cm}^2 \end{aligned}$$

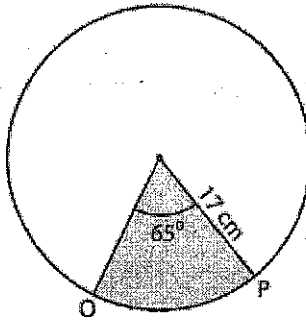
Find the length of the arc and area of the shaded region. Round the answer to two decimal places. (use  $\pi = 3.14$ )

1)



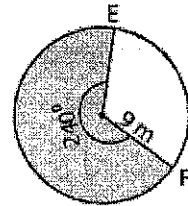
$$\begin{aligned} \text{A.L.} &= \frac{270 \cdot 2 \cdot \pi \cdot 12}{360} \\ \text{A} &= \frac{270 \cdot \pi \cdot 12^2}{360} \end{aligned}$$

2)



$$\begin{aligned} L &= \frac{65 \cdot 2 \cdot \pi \cdot 17}{360} \\ A &= \frac{65 \cdot \pi \cdot 17^2}{360} \end{aligned}$$

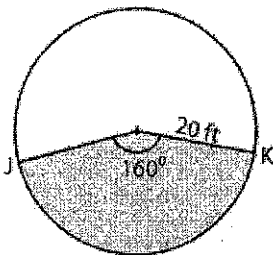
3)



$$\begin{aligned} L &= \frac{240 \cdot 2 \cdot \pi \cdot 9}{360} \\ A &= \frac{240 \cdot \pi \cdot 9^2}{360} \end{aligned}$$

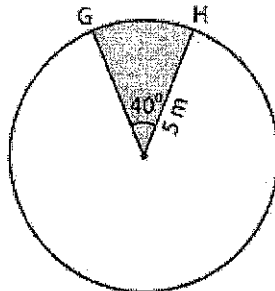
Length of the arc MN = 56.5 in    Length of the arc OP = 19.3 cm    Length of the arc EF = 37.7 m  
 Area of a sector = 339.3 in<sup>2</sup>    Area of a sector = 163.9 cm<sup>2</sup>    Area of a sector = 169.6 m<sup>2</sup>

4)

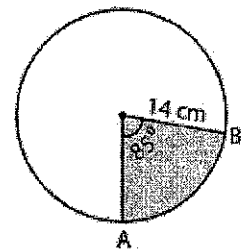


Length of the arc JK = 55.85 ft    Length of the arc GH = 3.49 m    Length of the arc AB = 20.8 cm  
 Area of a sector = 558.50 ft<sup>2</sup>    Area of a sector = 8.73 m<sup>2</sup>    Area of a sector = 145.39 cm<sup>2</sup>

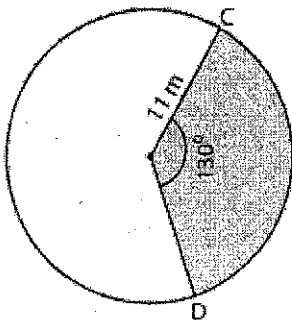
5)



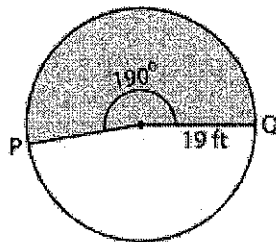
6)



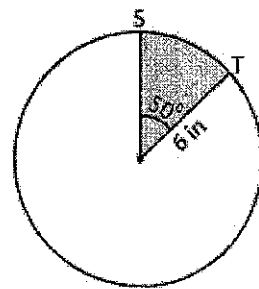
7)



8)



9)

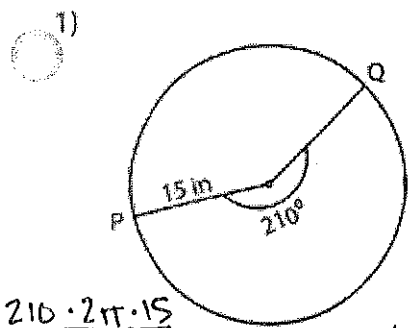


Length of the arc CD = 24.96 m    Length of the arc PQ = 63.01 ft    Length of the arc ST = 5.24 in  
 Area of a sector = 137.27 m<sup>2</sup>    Area of a sector = 598.56 ft<sup>2</sup>    Area of a sector = 15.71 in<sup>2</sup>



Find the missing one. Round the radius and central angle to the nearest whole number.

Round the arc length to two decimal places. (use  $\pi = 3.14$ )

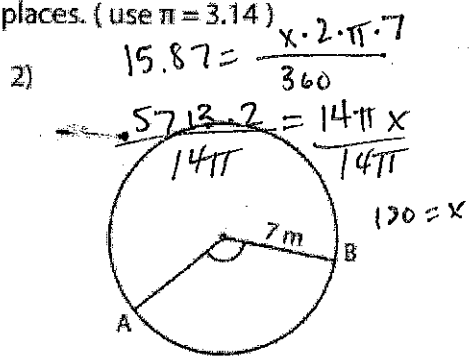


$$\frac{210 \cdot 2\pi \cdot 15}{360}$$

Radius = 15 in

Central angle = 210°

Length of the arc PQ = 54.98 in



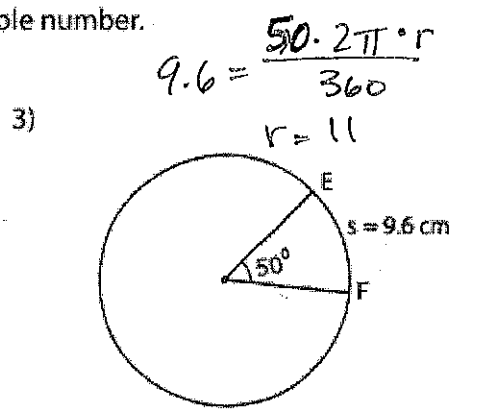
$$15.87 = \frac{x \cdot 2\pi \cdot 7}{360}$$

$$5713.2 = 14\pi x$$

Radius = 7 m

Central angle = 130°

Length of the arc AB = 15.87 m



$$9.6 = \frac{50 \cdot 2\pi \cdot r}{360}$$

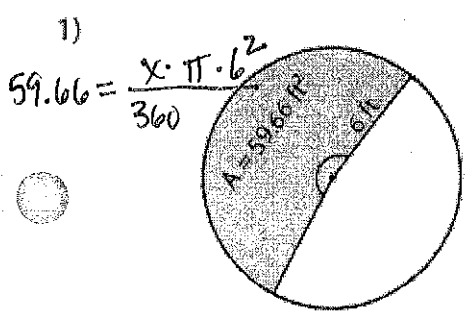
Radius = 11 cm

Central angle = 50°

Length of the arc EF = 9.6 cm

Find the missing one. Round the radius and central angle to the nearest whole number.

Round the area to two decimal places. (use  $\pi = 3.14$ )

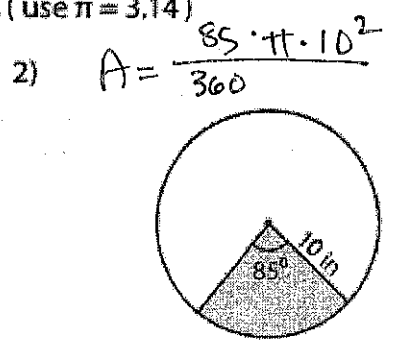


$$59.66 = \frac{x \cdot \pi \cdot 6^2}{360}$$

Radius = 6 ft

Central angle = 190°

Area of a sector = 59.66 ft<sup>2</sup>

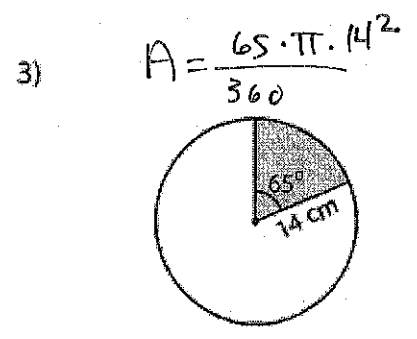


$$A = \frac{85 \cdot \pi \cdot 10^2}{360}$$

Radius = 10 in

Central angle = 85

Area of a sector = 74.18 in<sup>2</sup>



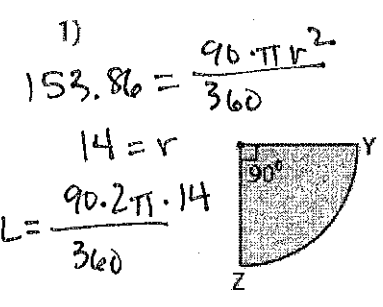
$$A = \frac{65 \cdot \pi \cdot 14^2}{360}$$

Radius = 14 cm

Central angle = 65°

Area of a sector = 111.18 cm<sup>2</sup>

Find the arc length for each sector. Round the answer to two decimal places. (use  $\pi = 3.14$ )



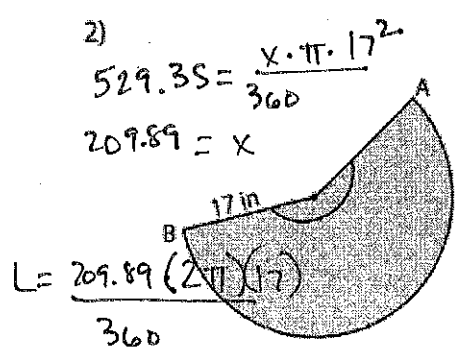
$$153.86 = \frac{90 \cdot \pi \cdot r^2}{360}$$

$$14 = r$$

$$L = \frac{90 \cdot 2\pi \cdot 14}{360}$$

Area = 153.86 cm<sup>2</sup>

Length of the arc YZ = 21.99 cm



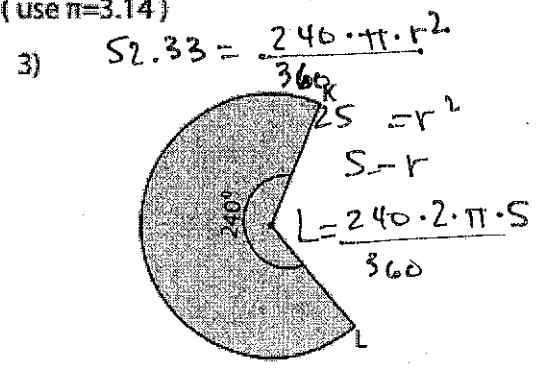
$$529.35 = \frac{x \cdot \pi \cdot 17^2}{360}$$

$$209.89 = x$$

$$L = \frac{209.89 \cdot (2\pi \cdot 17)}{360}$$

Area = 529.35 in<sup>2</sup>

Length of the arc AB = 62.28 in



$$52.33 = \frac{240 \cdot \pi \cdot r^2}{360}$$

$$25 = r$$

$$s = r$$

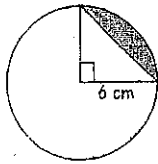
$$L = \frac{240 \cdot 2\pi \cdot 25}{360}$$

Area = 52.33 m<sup>2</sup>

Length of the arc KL = 20.94 m

Ex4: Find the area of the shaded region. Give answers in exact form unless otherwise stated.

a.

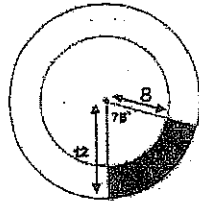


Sector - Triangle

$$\frac{90}{360} \cdot \pi \cdot 6^2 - \frac{1}{2}(6)(6)$$

$$9\pi - 18$$

b.



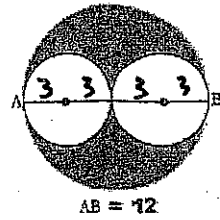
Lg Sector - Sm Sector

$$\frac{75}{360} \cdot \pi \cdot 12^2 - \frac{75}{360} \cdot \pi \cdot 8^2$$

$$30\pi - 13\frac{1}{3}\pi$$

$$16\frac{2}{3}\pi$$

c.



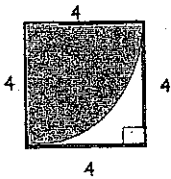
Big Circle - 2 Little Circles

$$\pi \cdot 6^2 - 2(\pi \cdot 3^2)$$

$$36\pi - 18\pi$$

$$18\pi$$

d.

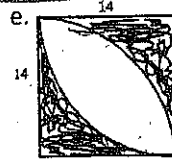


Square -  $\frac{1}{4}$  Circle

$$4^2 - \frac{1}{4} \pi \cdot 4^2$$

$$16 - 4\pi$$

e.



(round to the nearest tenth)

2 (Square -  $\frac{1}{4}$  Circle)

$$2(14^2 - \frac{1}{4} \cdot \pi \cdot 7^2)$$

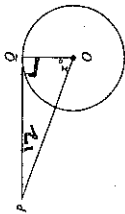
$$2(196 - 12.25\pi)$$

$$315.0$$

Common Core Unit 8 Test Review

Multiple Choice: Identify the choice that best completes the statement or answers the question.

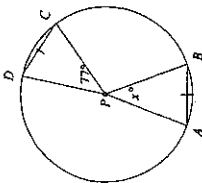
- A 1. Assume that lines that appear to be tangent are tangent.  $O$  is the center of the circle. Find the value of  $x$  if  $m\angle P = 12$ . (figures are not drawn to scale.)



$90 - 12$

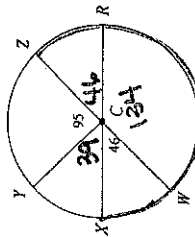
- a. 78      b. 39      c. 102      d. 24

- C 2. Find the value of  $x$ . If necessary, round your answer to the nearest tenth. The figure is not drawn to scale.



- a. 13      b. 26      c. 77      d. 38.5

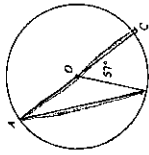
- D 3.  $\overline{WZ}$  and  $\overline{XR}$  are diameters. Find the measure of arc  $ZHX$ . (The figure is not drawn to scale.)



$$\begin{array}{r} 360 \\ - 95 \\ - 46 \\ \hline \end{array}$$

- a. 226      b. 275      c. 39      d. 321

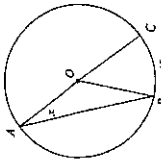
- B 4. Find the measure of  $\angle BAC$ . (The figure is not drawn to scale.)



$57 \div 2 = 28.5$

- a. 57      b. 28.5      c. 33      d. 114

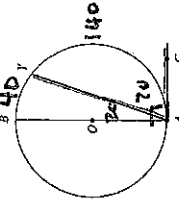
- C 5. Find  $x$ . (The figure is not drawn to scale.)



$\frac{1}{2}(46)$

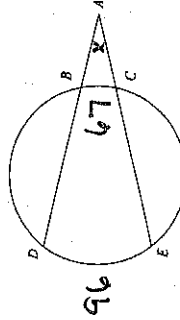
- a. 92      b. 44      c. 23      d. 46

- C 6. If  $m(\text{arc } BY) = 40$ , what is  $m\angle YAC$ ? (The figure is not drawn to scale.)



- a. 140      b. 100      c. 70      d. 80

- A 7.  $m(\text{arc } DE) = 96$  and  $m(\text{arc } BC) = 67$ . Find  $m\angle A$ . (The figure is not drawn to scale.)

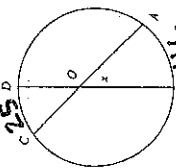


$$\frac{96 - 67}{2} = 29$$

- a. 14.5      b. 62.5      c. 81.5      d. 29

8. Find the value of  $x$  for  $m(\text{arc } AB) = 46$  and  $m(\text{arc } CD) = 25$ . (The figure is not drawn to scale.)

$$\frac{25 + 46}{2} = \frac{61}{2} = 30.5$$



- a.  $35.5^\circ$       c.  $71^\circ$       d.  $21^\circ$   
 b.  $58.5^\circ$
9. Write the standard equation for the circle with center  $(2, 7)$ ,  $r = 4$
- a.  $(x-7)^2 + (y-2)^2 = 16$       c.  $(x-2)^2 + (y-7)^2 = 16$   
 b.  $(x-2)^2 + (y-7)^2 = 4$       d.  $(x+2)^2 + (y+7)^2 = 4$

10. Write the standard equation for the circle with center  $(-6, -8)$ , that passes through  $(0, 0)$

a.  $(x-6)^2 + (y-8)^2 = 10$   
 b.  $(x-6)^2 + (y-8)^2 = 196$   
 c.  $(x+6)^2 + (y+8)^2 = 14$   
 d.  $(x+6)^2 + (y+8)^2 = 100$

$$d = \sqrt{(0+6)^2 + (0+8)^2}$$

$$= \sqrt{6^2 + 8^2}$$

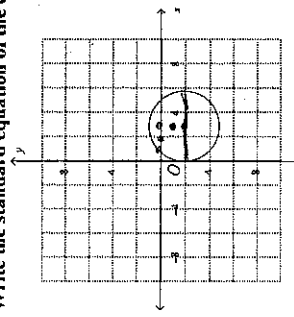
$$= \sqrt{100}$$

10

11. Find the center and radius of the circle with equation  $(x+9)^2 + (y+5)^2 = 64$ .

- a. center  $(5, 9)$ ;  $r = 8$   
 b. center  $(9, 5)$ ;  $r = 64$   
 c. center  $(-9, -5)$ ;  $r = 64$   
 d. center  $(-9, -5)$ ;  $r = 8$

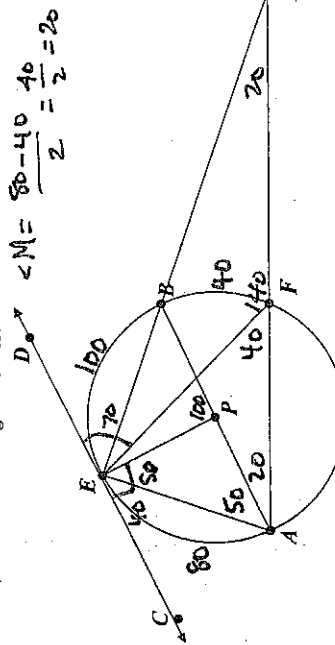
12. Write the standard equation of the circle in the graph.



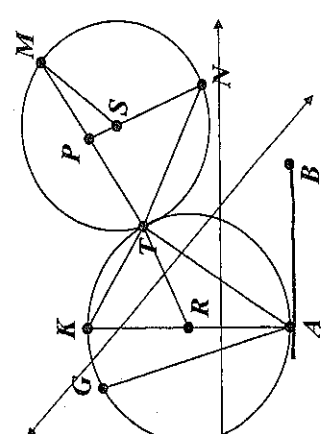
- a.  $(x+3)^2 + (y-2)^2 = 9$   
 b.  $(x-3)^2 + (y+2)^2 = 9$   
 c.  $(x-3)^2 + (y+2)^2 = 18$   
 d.  $(x+3)^2 + (y-2)^2 = 18$

In the figure,  $\overline{AB}$  is a diameter, P is the center of the circle,  $\overline{CD}$  is a tangent to the circle at E. If  $m\widehat{BE} = 100^\circ$  and  $m\widehat{BF} = 40^\circ$ , find the following measures:

13.  $\angle AEP$   
 14.  $\angle AEF$   
 15.  $\angle CEA$   
 16.  $\angle EPB$   
 17.  $\angle M$   
 18.  $\angle EAB$   
 19.  $\angle EFA$   
 20.  $\angle PED$   
 21.  $\angle AEP$   
 22.  $\angle EFM$   
 23.  $\angle DEF$   
 24.  $\angle BAF$



Matching. In the figure the two circles, with centers R and S, intersect only at T and  $\overline{AB} \perp \overline{RA}$ .



25.  $\overline{AB}$  is a tangent.  
 26.  $\overline{KA}$  is a radius.  
 27.  $\overline{NS}$  is a radius.  
 28.  $\overline{BG}$  is a secant.  
 29. Circles R and S are externally tangent.  
 30.  $\overline{KT}$  is a chord of circle.  
 31. R is a center of circle.  
 32. Point P is a(n) interior point of circle S.  
 33. Point B is a(n) exterior point of circle S.

- A. diameter  
 B. chord  
 C. secant  
 D. radius  
 E. center of circle  
 F. tangent  
 G. exterior point  
 H. interior point  
 I. externally  
 J. internally

34. In a circle with radius 6, a sector has an area  $15\pi$ . What is the length of the arc of the sector?

Length of the arc =  $\frac{5\pi}{360} \times 2\pi r$   
 $\approx 15.7$        $A = \frac{x \cdot \pi r^2}{360} = 15\pi$

35. The radius of a sector is 12 and the measure of the arc is  $130^\circ$ . What is the length of the arc and the area of the sector?

a)  $\frac{27.2}{360} \times 2\pi \cdot 12$   
 b)  $\frac{163.4}{360} \times 2\pi \cdot 12$

A.L. =  $\frac{x \cdot \pi r^2}{360} = \frac{130 \cdot \pi \cdot 12^2}{360} = 150.2\pi$   
 $\approx 163.4$

A.L. =  $\frac{x \cdot \pi r^2}{360} = \frac{130 \cdot \pi \cdot 12^2}{360} = 150.2\pi$   
 $\approx 163.4$