Polynomials Test Review

Degree of a monomial: sum of the exponents of the variables

Degree of a polynomial: the highest monomial degree in the expression

Exponent Rules:

Multiplying same bases: $a^m \cdot a^n = a^{m+n}$

Dividing Exponents: $\frac{a^m}{a^n} = a^{m-n}$

Power to a power: $(a^m)^n = a^{m \cdot n}$

Negative exponents: $a^{-n} = \frac{1}{a^n}$ $\quad \frac{1}{a^{-n}} = a^n$

* Never leave negative exponents! *

Factoring

Always, Always, Always! Pull out a GCF!

Difference of Squares: $a^2 - b^2 = (a-b)(a+b)$

Sum of Cubes: $(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$

Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

For sum/diff of cubes, to determine signs, remember "SOP" → same, opposite, positive

Polynomial Test Review

Perfect Square Trinomial

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

For other trinomials, use other method: guess and check, box, etc.

When adding/subtracting polynomials, combine like terms

ex $(x^2 + x - 3) - (2x^2 - x - 3)$
$$= x^2 + x - 3 - 2x^2 + x + 3$$
$$= -x^2 + 2x$$

When multiplying, distribute terms

$$(x+2)(x+3) = x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

$$x(x^2 + 3x) = x^3 + 3x^2$$

$$(x+2)(x^2 + x + 2) = x^3 + x^2 + 2x + 2x^2 + 2x + 4$$
$$= x^3 + 3x^2 + 4x + 4$$

When multiplying monomials... multiply like terms
$$(4xy^2)(-2x^3y^3) = -8x^4y^5$$

* When multiplying radicals, multiply radicands *

Rules for Working with Square Roots

index $\rightarrow$ $\sqrt[n]{x}$ $\leftarrow$ radicand

I. To Simplify:

① The radicand <u>cannot</u> contain a Factor that is a perfect square

ex) $\sqrt{125} = \sqrt{25 \cdot 5} = 5\sqrt{5}$

ex) $\sqrt{9x^2y^4z^5}$

$= 3xy^2z^2\sqrt{z}$ (* divide exponents by 2)

② The denominator <u>cannot</u> contain a radical (see III).

II. To Add/Subtract Square Roots:

① Simplify radicals 1st (see I above).

② You can only +/− radicals that have the same indices <u>and</u> radicands.

ex) $3\sqrt{2} + \sqrt{50}$

$= 3\sqrt{2} + \sqrt{25 \cdot 2}$

$= 3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$

(* Add only coefficients)

III. To Divide Square Roots:

① If denominator has 1 radical, multiply by itself: $\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{6\sqrt{2}}{2}$

$= 3\sqrt{2}$

② If den is a binomial, multiply by conjugate:

$\frac{6}{3+\sqrt{2}} \cdot \frac{3-\sqrt{2}}{3-\sqrt{2}}$

$= \frac{18-6\sqrt{2}}{9-2} = \frac{18-6\sqrt{2}}{7}$

Polynomials Test Review

Imaginary Numbers

$i = \sqrt{-1}$

When working with a negative under the radical, always pull out "i" first!

$i = i$	$i^5 = i$	This pattern repeats forever.
$i^2 = -1$	$i^6 = -1$	To simplify, divide exponent
$i^3 = -i$	$i^7 = -i$	by 4. Raise "i" to your
$i^4 = 1$	$i^8 = 1$	remainder to simplify

☆ Never leave a power > 1 ☆

Always put imaginary numbers in standard form

$a + bi$

When adding, combine like terms
and subtracting

When multiplying, follow distributive prop.

When dividing, never leave i in the denominator!

either multiply by $\frac{i}{i}$ or the

conjugate

The conjugate of $a+bi$ is $a-bi$
where a and b are numbers.

☆ Remember to substitute -1 in for i^2 ☆

☆ Imaginary numbers come in pairs! ☆

Polynomials Test Review

Completing the Square

1. Isolate variables and constants
2. Factor out x^2 coefficient if not 1.
3. Complete the square by adding $(b/2)^2$ to each side
4. Factor (of the form $(x+b/2)^2$)
5. Take the square root of both sides, $\pm\sqrt{}$ of constant!
6. Solve for x.

★ To put quadratics in vertex form, complete the square (isolate "x" terms) and then solve for y.

$y = a(x-h)^2 + k$ (h, k) is vertex

ex: $y = -2x^2 - 24x - 75$

$y + 75 = -2x^2 - 24x$ isolate x terms

$y + 75 = -2(x^2 + 12x)$ factor out leading coef.

$y + 75 + (-72) = -2(x^2 + 12x + 36)$ complete the $\square$

$y + 3 = -2(x+6)^2$ simplify / factor

$y = -2(x+6)^2 - 3$ solve for y

Solving quadratics with quad formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

discriminant $= b^2 - 4ac$

★ no radical on d! ★

★ put quadratics in standard form ★

4 cases:

- $d = 0 \rightarrow$ 1 real rational double root
- $d < 0 \rightarrow$ 2 imaginary roots
- $d > 0$ and perfect square $\rightarrow$ 2 real rational roots
- $d > 0$ and NOT a perfect square $\rightarrow$ 2 real irrational roots

Polynomials Test Review

Focus and Directrix

The vertex is equidistant from the focus and directrix

The vertex is always in between f + d

The focus is always inside of the parabola

If a is negative, parabola opens down

$a = \frac{1}{4p}$ where p is the distance from vertex to f and vertex to d

* The focus is always a point
* The directrix is always a horizontal line $y=c$

End Behavior: Look at page 48 (chart)

* Review List on page 49

* imaginary numbers ALWAYS come in conjugate pairs *
i.e. if $9+i$ is a root $\Rightarrow$ $9-i$ is also a root

* Practice long and synthetic division
remember to use placeholders
$4x^4+2x^2+3 \rightarrow 4x^4+0x^3+2x^2+3$

* Remainder Theorem: If the remainder is 0 $\Rightarrow$ the divisor is a root of the polynomial