

Perform the indicated operations:

$$1.) \quad 2(3a^2 + a - 6) + (-a - a^2 + 1)$$

$$\begin{array}{r} 6a^2 + 2a - 12 \\ -a^2 - a + 1 \\ \hline 5a^2 + a - 11 \end{array}$$

$$2.) \quad (5a^3 - 4a^2 - a) - 3(a + 6 - 2a^2)$$

$$\begin{array}{r} 5a^3 - 4a^2 - a \\ -3a - 18 + 6a^2 \\ \hline 5a^3 + 2a^2 - 4a - 18 \end{array}$$

$$3.) \quad \frac{3-2i}{9+9i} \cdot \frac{-i}{-i} = \frac{-3i+2i^2}{-9i^2} = \frac{-2-3i}{9}$$

$$4.) \quad 3i(2+i)(3-4i)$$

$$\begin{array}{l} (6i+3i^2)(3-4i) \\ (6i-3)(3-4i) \end{array}$$

$$\begin{array}{r} 18i - 24i^2 - 9 + 12i \\ 15 + 30i \end{array}$$

$$4a) \quad \frac{(4+3i)(2+3i)}{2-3i} = \frac{8+9i^2+12i+6i}{4+9} = \frac{-1+18i}{13}$$

$$4b) \quad 2i^{84} - 2i^{107}$$

$$2(1) + 2i$$

$$2 + 2i$$

$$4 \sqrt[3]{107}$$

$$\begin{array}{r} 26 \\ 8 \\ \hline 27 \\ 24 \\ \hline 3 \\ i^3 = -i \end{array}$$

Solve:

$$5.) \quad x^3 - 9x^2 + 20x = 0$$

$$x(x^2 - 9x + 20) = 0$$

$$x(x-4)(x-5) = 0$$

$$\{0, 4, 5\}$$

$$6.) \quad (x^2 + 9)^4(x^2 - 5) = 0$$

$$x^2 = -9 \quad x^2 = 5$$

$$x = \pm 3i \quad x = \pm \sqrt{5}$$

$$\text{mult: } 4 \quad \{ \pm 3i \text{ mult: } 4, \pm \sqrt{5} \}$$

$$7.) \quad 3x^2(x+12)^2(x-5) = 0$$

$$\{0 \text{ mult: } 2, -12 \text{ mult: } 2, 5\}$$

$$8.) \quad x^4 + 9x^2 + 20 = 0$$

$$(x^2 + 4)(x^2 + 5) = 0$$

$$x^2 = -4 \quad x^2 = -5$$

$$x = \pm 2i \quad x = \pm i\sqrt{5}$$

$$\{ \pm 2i, \pm i\sqrt{5} \}$$

Given:

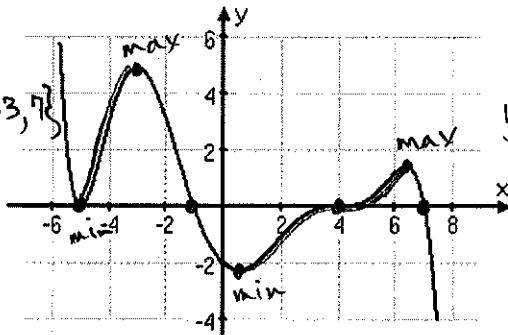
9.)

Roots & multiplicity if any:
 $\{-5 \text{ mult: } 2, -1, 4 \text{ mult: } 3, 7\}$

Domain (interval notation)
 $(-\infty, \infty)$

Maximum (local)
 $(-3, 5)$ and $(6.5, 1.5)$

End behavior:
 $x \rightarrow \infty \quad y \rightarrow -\infty$
 $x \rightarrow -\infty \quad y \rightarrow \infty$



write in terms of its linear factors:
 $y = (x+5)^2(x+1)(x-4)^3(x-7)$

Range (interval notation) $(-\infty, \infty)$

Increasing: $(-5, -3) \cup (1.5, 6.5)$

minimum: $(-5, 0)$ and $(1.5, -2.25)$

Given the roots of a polynomials function write the function is standard form. **MUST SHOW WORK!**

10.) $x = -1$ with multiplicity of 3

$$y = (x+1)^3 = (x+1)(x+1)(x+1)$$

$$y = (x^2 + 2x + 1)(x+1)$$

$$y = x^3 + x^2 + 2x^2 + 2x + x + 1$$

$$y = x^3 + 3x^2 + 3x + 1$$

11.) $x = 7$ and $x = -2i \quad x = 2i$

$$y = (x-7)(x+2i)(x-2i)$$

$$y = (x-7)(x^2+4)$$

$$y = x^3 - 7x^2 + 4x - 28$$

12.) Given: $f(x) = x^4 + 2x^3 + 5x^2 + 8x + 4$ with a root of $x = -1$ with a multiplicity of two, find the

remaining solutions:

$$\begin{array}{r|rrrrr} -1 & 1 & 2 & 5 & 8 & 4 \\ & \downarrow & -1 & -1 & -4 & -4 \\ \hline & 1 & +1 & 4 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & 4 & 4 \\ & \downarrow & -1 & 0 & -4 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$$\begin{aligned} x^2 + 4 &= 0 \\ \sqrt{x^2} &= \sqrt{-4} \\ x &= \pm 2i \end{aligned}$$

* 13.) List all possible rational roots using the rational root theorem: SHOW YOUR WORK

$$y = 6x^4 - 2x^2 + 2x - 9$$

$$\begin{aligned} p &= 9 & \pm 1, \pm 3, \pm 9 \\ q &= 6 & \pm 1, \pm 2, \pm 3, \pm 6 \end{aligned}$$

$$\text{Possibles: } \pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$$

14.) One factor of $x^3 + 3x^2 + x + 3$ is $(x + 3)$ factor completely:

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 1 & 3 \\ & \downarrow & -3 & 0 & -3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$x^2 + 1 = 0$$

$$\sqrt{x^2} = \sqrt{-1}$$

$$x = \pm i$$

$$(x+3)(x+i)(x-i) \leftarrow \text{or}$$

15.) Divide using long division: $(3x^4 + 2x^3 - 8x - 48)$ by $(x^2 - 4)$

$$\begin{array}{r} 3x^2 + 2x + 12 \\ x^2 - 4 \overline{) 3x^4 + 2x^3 + 0x^2 - 8x - 48} \\ \underline{-3x^4} \\ 2x^3 + 12x^2 - 8x - 48 \\ \underline{-2x^3} \\ 4x^2 - 8x - 48 \end{array}$$

$$\begin{array}{r} 12x^2 - 48 \\ \underline{12x^2 - 48} \\ 0 \end{array}$$

16.) Find $p(-2)$ when $p(x) = 3x^4 + 5x^3 + x - 3$

$$\begin{aligned} p(-2) &= 3(-2)^4 + 5(-2)^3 + (-2) - 3 \\ &= 48 - 40 - 5 \end{aligned}$$

$$p(-2) = 8 - 5 = 3$$

Check:

$$\begin{array}{r|rrrrr} -2 & 3 & 5 & 0 & 1 & -3 \\ & \downarrow & -6 & 2 & -4 & 6 \\ \hline & 3 & -1 & 2 & -3 & 3 \end{array}$$

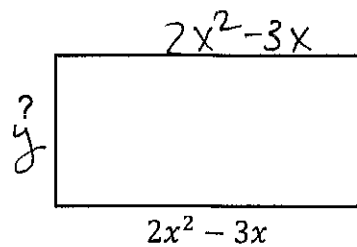
17) divide $9x^{102} - 7x^{51} + 12x^{31} - 2x^{22} + 6$ by $(x + 1)$ what will be the remainder?

$$p(-1) = 8$$

$$\begin{array}{r|rrrrrrrrrrrr} -1 & 9 & 0 & 0 & -7 & 0 & 0 & 0 & 12 & 0 & 0 & -2 & 0 & 6 \\ & \downarrow & -9 & 9 & -9 & 16 & -16 & 16 & -16 & 4 & -4 & 4 & -2 & 2 \\ \hline & 9 & -9 & 9 & -16 & 16 & -16 & 16 & -4 & 4 & -4 & 2 & -2 & 8 \end{array}$$

18) Perimeter is $26x^2 - 12x + 4$, find the length of the missing side?

$$\begin{aligned} 26x^2 - 12x + 4 &= 2(2x^2 - 3x) + 2y \\ &= 4x^2 - 6x + 2y \\ \underline{-4x^2 + 6x} & \\ 2y &= 4 \end{aligned}$$



$$\frac{2y}{2} = \frac{4}{2} \implies y = 2$$

$$y = 11x^2 - 3x + 2$$