Honors Math 3 ~ Spring 2018 Name______________________

Unit 3: Polynomial Functions

Date	Lesson	Assignment
Monday 2/12	Introduction to Polynomials p. 1 - 7	
Tuesday 2/13	Factoring Review p. 8 - 13	
Wednesday 2/14	Polynomial Graphs and Key Features p.14 - 20	
Thursday 2/15	Transformations & Graphs of Polynomials p. 21 - 27	
Friday 2/16	Polynomial Equations and Models p. 28 - 32 STEM Project (due Monday, March 5th)	
Monday 2/19	**Quiz** Long Division p. 33 - 36	
Tuesday 2/20	Synthetic Division p. 36 - 39	
Wednesday 2/21	Long Division & Synthetic Division p. 40 - 42	
Thursday 2/22	Rational Root Theorem p. 43 - 45 **Quiz**	
Friday 2/23	Analyzing Functions 46 - 51	
Monday 2/26	Review p. 52 -56	
Tuesday 2/27	**Test**	

VOCABULARY

- A **quadratic function** is a function of the form $f(x) = ax^2 + bx + c$ where $a,b,\&c$ are real numbers and $a \neq 0$.
- **Standard form** of a quadratic equation $y = ax^2 + bx + c$
- **Vertex form** of a quadratic equation $y = a(x-h)^2 + k$
- The **vertex** of a quadratic equation is the maximum or minimum point.
- **Zeros, solutions, roots** are the values of x that make y=0.
- **x-intercepts** are the points where a graph crosses the x-axis, the real values of x that make y=0.
- The **quadratic formula** is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ It can be used to solve a quadratic in standard form.
- A **complex number** is any number that can be written in the form **a + b*i***, where a and b are real numbers, and $i = \sqrt{-1}$.
- The **complex conjugate** of *a + bi* is *a - bi* .
- A ***monomial*** is an expression that is a real number, a variable, or the product of real numbers and variables.
- A ***polynomial*** is a monomial or the sum of monomials.
- A ***polynomial function*** is any function with a rule that can be written in the form: $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ where $a_n \neq 0$, coefficients ($a_n, \ldots, a_0$) are real numbers, and exponents are nonnegative integers.
- Any algebraic expression in the form $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ is called a ***polynomial expression***.
- The **degree** of a polynomial f(x) is the highest exponent on the variable, x.
- The ***leading coefficient*** of a polynomial is the coefficient of the term with the highest degree.
- In ***standard form*** the terms of a polynomial are written in descending order by degree.
- **End behavior** Is the behavior of a graph as x approaches $+\infty$ or $-\infty$.
- **Multiplicity** is the number of times a zero occurs.
- **Synthetic division** is a shortcut method for dividing a polynomial by a linear factor of the form *(x – a)*.

Day 1 Polynomial Operations Warm Up

1. $(19x^2 + 12x + 12) + (7x^2 + 10x + 13)$

2. $(19x^2 + 9x + 16) - (5x^2 + 12x + 7)$

3. $-x^2(x + 5)$

4. $3x(-x^2 + 2x - 12)$

5. $(4x - 3)(5x + 4)$

6. $(2x - 3)(4x^2 + 8x - 2)$

Unit 3 Polynomials

Introduction to Polynomials; Polynomial Graphs and Key Features

All Polynomials must have whole numbers as exponents!!

Example: $9x^{-1} + 12x^{\frac{1}{2}}$ is NOT a polynomial.

Polynomial Vocabulary Review

- **Expression:**
- **Equation:**
- **Terms:**
  - **Monomial, Binomial, Trinomial, Polynomial**
- **Degree:**
  - **Constant, Linear, Quadratic, Cubic, Quartic**

Example 1: Fill in the table below.

POLYNOMIAL	NUMBER OF TERMS	CLASSIFICATION BY TERMS	DEGREE	CLASSFICATION BY DEGREE	SKETCH THE GRAPH
$f(x) = 5$					
$g(x) = 4x - 3$					
$p(x) = -2x^5$					
$w(x) = x^4 - 4x + 2$					
$y = -4x^2 + x + 9$					
$h(x) = 4x^3 + x^2 - 9x + 2$					

Math 3
Polynomial Notes

Name: ____________
Date : ____________

When we write polynomials we like for the terms to be in descending order of the powers of the variables.

Example:

$2x + x^2 + 1 \longrightarrow x^2 + 2x + 1$

$3y^2 + 5y^3 + y \longrightarrow 5y^3 + 3y^2 + y$

$x^2 + y^2 + 3xy \longrightarrow x^2 + 3xy + y^2$

Degree:

Degree of a Monomial: The sum of the exponents of the variables.
Degree of a Polynomial: The highest monomial degree in the expression.

Examples:

Polynomial	Terms	Degree of the terms	Degree of the polynomial
2x+7	2x,7	1,0	1
$3x^2 + 5x$	$3x^2, 5x$	2,1	2
$a^6 + 2a^3 + 1$	$a^6, 2a^3, 1$	6,3,0	6
$5x^4 - 4a^2b^6 + 3x$	$5x^4, -4a^2b^6, 3x$	4,8,1	8

Exponent Rules for multiplying and Dividing:

Multiplying same base: $a^m \cdot a^n = a^{m+n}$

Power to a power: $(a^m)^n = a^{mn}$

Dividing Exponents: $\frac{a^m}{a^n} = a^{m-n}$

Negative Exponents: $a^{-n} = \frac{1}{a^n}$ or $\frac{1}{a^{-n}} = a^n$

Examples:

1. $(2x^2y)^3(xy)^3$

Power to power: $(2^3 x^{2*3} y^{1*3})(x^{1*3} y^{1*3})$

$(8x^6y^3)(x^3y^3)$

Multiply same bases: $8x^{6+3}y^{3+3}$

$8x^9y^6$

2. $\frac{18x^2y^3}{6x^5y}$

Reduce constants and subtract exponents.

$\left(\frac{18}{6}\right)(x^{2-5})(y^{3-1})$

$(3)(x^{-3})(y^2)$

Negative exponents go to the denominator

$\frac{3y^2}{x^3}$

CW: Polynomials

Name: ____________________
Date: ____________________

I. Write each polynomial in descending powers of the variable.

1. $14x+2-3x^2+5x^3$ ____________________
2. $8z^2-2z+7-9z^3$ ____________________
3. $8x^3-3x+2x^2-5$ ____________________
4. $7z^2+2-6z^4+5z$ ____________________
5. $16y+2y^5-1+10y^2$ ____________________

II. Classify each as a monomial, binomial, or trinomial.

6. $4x^4-9xy$ ____________________
7. $7xy+2x-6y$ ____________________
8. $3x^2+2x-6$ ____________________
9. $\frac{1}{3}y+6x$ ____________________
10. $-3x^4y^5z^9$ ____________________
11. -2 ____________________

III. Find the degree of each monomial

12. x^7y^5 ____________________
13. $x^2y\ z^3$ ____________________

IV. Simplify each polynomial and write the result in order of descending powers. Then give the degree of the polynomial.

14. $7x^3+6x^2-9x^5-4x^2$ ____________________ ______
15. $-3+2x+5x^2+9-6x+10x^2$ ____________________ ______

VII. Add or subtract the following expressions. Write the result in order of descending powers.

16. $(-2t-3r+s)+(3t-5r-2s)=$ ____________________
17. $(a+10e-f)+(2a+4f)=$ ____________________
18. $(x^3-6x^2-7)+(-3x^3+2x^2-9)=$ ____________________
19. $(3a^2+5b^2)+(6a^2-3ab+b^2)+(2a^2+7ab)=$ ____________________
20. $(4x^2+5xy-3y^2)-(6x^2+8xy+3y^2)=$ ____________________
21. $(6x^2+9x-3)-(4x^2+11x-7)=$ ____________________
22. $(m^2+10m-13)-(m^2-7m-2)=$ ____________________
23. $(2x-9)+(6x-7)-(3x+1)=$ ____________________
24. $(11a^2-7a+1)+(2a^2-9a+13)-(17a^2+2a-11)=$ ____________________
25. $(19x^2+7x-2)+(4x+11x^2-7)-(15+5x^2-9x)=$ ____________________
26. $(9x^2-2)-(7x+5)+(3x-2)=$ ____________________

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V. Multiply

22. $(-2rs)(5r^2)(s^2)=$ ________________

23. $(-2m^3)^3=$ ________________

24. $(2a^4)(2a^3b^2)(-3ab^3)=$ ________________

25. $(4a^3b^2c)^2(-ab^3c^2)^2=$ ________________

26. $(-2x)(-5x^4y^2)(-xy^5)=$ ________________

27. $(-3ab^2c)(-2a^2b^3c^2)^2(-5a^3bc^5)=$ ________________

28. $2a(2a^2+5a+1)=$ ________________

29. $(3x^5y^{-2})(-7x^{-10}y^5)=$ ________________

30. $(3x^2y)(3x^2y^3-2xy^4-5)=$ ________________

31. $(6x^3y^2)^2(3x^{-3}y^{-7})=$ ________________

VI. Simplify

32. $\dfrac{16xy^3z^2}{4x^2yz^3}=$ ____________

33. $\dfrac{24r^5s^2t^2}{36r^3s^2t}=$ ____________

34. $\dfrac{-8a^4b}{-16ab^4}=$ ____________

35. $\dfrac{m^2}{m^{-4}}=$ ____________

36. $\dfrac{-9m^3n^{-5}}{27m^{-2}n^2}=$ ____________

37. $\dfrac{x^{-4}y^{-6}}{x^2y^{-9}}=$ ____________

VII. Multiply using the Box Method

38. $(9x+4)(3x+8)$

39. $(3w+6)(w-4)$

40. $(4x+5)^2$

41. $(3y-6)(4y^2+2y-7)$

42. $(2x+3)(5x^2-4x+1)$

43. $(3x^2+2x-1)(x^2+2x-3)$

Greatest common factor (GCF)

Definition: Coefficient and variables that are common to all terms in a polynomial

Ex 1: $5x^4 + 20x^3$ GCF:

Factored Form:

Ex 2: $12x^2y - 20x^3y$ GCF:

Factored Form:

Ex 3: $10p^6q^2 - 4p^5q^3 - 2p^4q^4$ GCF:

Factored Form:

Practice

1. $5x^2y^3 + 15x^3y^2 =$
2. $3y^2 - 3y - 9 =$
3. $6ab - 4ad + 12ac =$
4. $9x^3y^6z^2 - 12x^4y^4z^4 + 15x^2y^5z^3 =$
5. $14a^4b^3c^5 + 21a^3b^5c^4 - 35a^4b^4c^3 =$
6. $-4m^4 - 32m^3 + 64m =$

Differences of Squares

Formula: $a^2 - b^2 = (a-b)(a+b)$

Ex 1: $x^2 - 49 =$

Ex 2: $4x^2 - 9y^8 =$

Ex 3: $12a^2 - 48 =$

Ex 4: $a^2 + 9 =$

Ex 5: $-45x^2 + 20y^2z^2 =$

Sums and Differences of Cubes

Formula: $a^3 - b^3 =$

$a^3 + b^3 =$

Ex 1: $x^3 - 8 =$

Try 2: $y^3 + 27 =$

Ex 3: $8x^3 - 27y^3 =$

Try 4: $64x^3 + 125y^3 =$

Name : ____________________ Score : ____________

Factors: Algebraic Identity

Factorize each polynomial using algebraic identity.

1) $4s^3t^2 - 36s$

2) $\frac{25}{9}u^2 - v^2$

3) $5k^2 - 125$

4) $3p^2 - 48q^2$

5) $x^2 - \frac{1}{9}$

6) $98a^2 - 2b^2$

7) $m^2n^2 - 49$

8) $64x^3 - 16xy^2$

10

 Name________________________

Factoring Trinomials (a = 1)

Date________________ Period____

Factor each completely.

1) $b^2 + 8b + 7$

2) $n^2 - 11n + 10$

3) $m^2 + m - 90$

Product : -90
Sum : 1

1·90
2·45
3·30
5·18
6·15
-9·10

(m-9)(m+10)

4) $n^2 + 4n - 12$

5) $n^2 - 10n + 9$

6) $b^2 + 16b + 64$

7) $m^2 + 2m - 24$

8) $x^2 - 4x + 24$

9) $k^2 - 13k + 40$

10) $a^2 + 11a + 18$

11) $n^2 - n - 56$

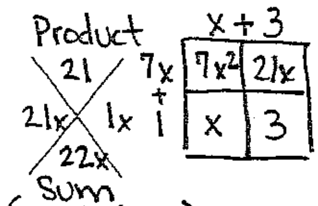
12) $n^2 - 5n + 6$

11

Algebra II
Factoring Trinomials

Name: ______________
Date: ______________

1. $7x^2 + 22x + 3$



(7x+1)(x+3)

2. $8a^2 - 10a + 3$

____

3. $3y^2 + 8y + 5$

____

4. $6x^2 - 11x + 4$

____

5. $2a^2 + 3a - 14$

____

6. $2x^2 - 5x - 12$

____

7. $2x^2 - 9x - 18$

____

8. $6a^2 + 19a + 10$

____

9. $3x^2 + 2x - 16$

____

10. $5x^2 - 22x + 8$

____

11. $6a^2 - 5a - 4$

____

12. $18x^2 + 9xz + z^2$

____

CCIII Honors
Factoring GCFs and Formulas

Name:____________________
Date:____________________

Factor COMPLETELY!

1. $15a^2b - 10ab^2 =$ ______________________________

2. $2x^3y - x^2y + 5xy^2 =$ ______________________________

3. $16r^2 - 169 =$ ______________________________

4. $c^2 - 49 =$ ______________________________

5. $2y^2 - 242 =$ ______________________________

6. $x^3 + 8 =$ ______________________________

7. $8m^3 - 1 =$ ______________________________

8. $b^4 - 81 =$ ______________________________

9. $2t^3 + 32t^2 + 128t =$ ______________________________

10. $x^2 - 8x - 16 =$ ______________________________

11. $4x^6 - 4x^2 =$ ______________________________

12. $81x^4 - 16 =$ ______________________________

13. $5y^5 + 135y^2 =$ ______________________________

14. $16r^2 - 24r + 9 =$ ______________________________

15. $4a^2 - 12a + 9 =$ ______________________________

Day 2 Quadratic Formula Warm Up

The Quadratic Formula: For quadratic equations: $ax^2+bx+c=0$,

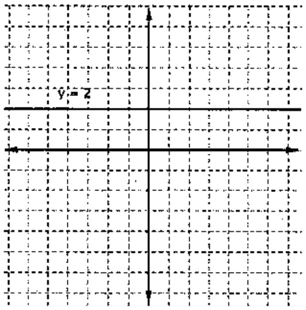
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Solve each equation using the Quadratic Formula.

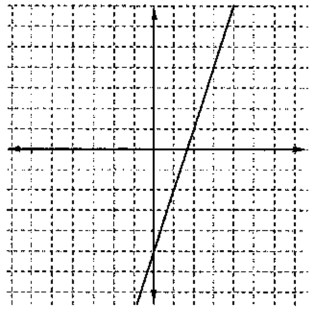
1. $4x^2+11x-20=0$
2. $x^2-5x-24=0$
3. $x^2=3x+3$
4. $x^2+5=-5x$
5. $x^2=-x+1$
6. $4x^2-1=-8x$

GRAPHS of POLYNOMIAL FUNCTIONS:

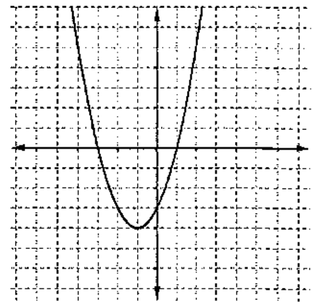
Constant function
$y = 2$
Degree 0



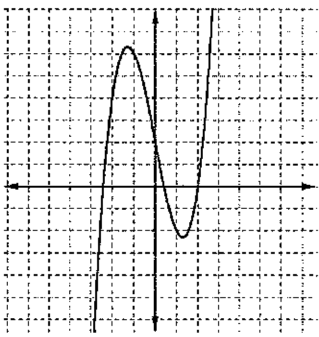
Linear function
$y = 3x - 5$
Degree 1



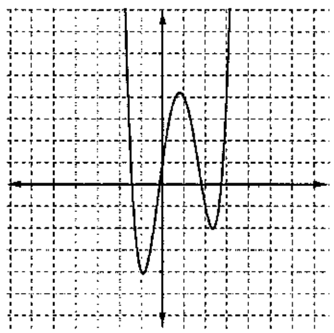
Quadratic function
$y = x^2 + 2x - 3$
Degree 2



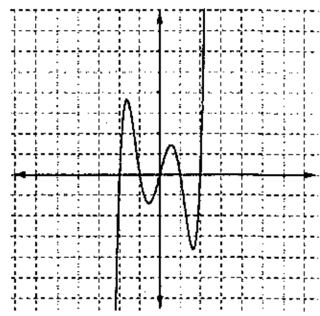
Cubic function
$Y = x^3 - 5x + 2$
Degree 3



Quartic function
$y = x^4 - 3x^3 - 2x^2 + 7x + 1$
Degree 4



Quintic function
$y = x^5 - 5x^3 + 4x$
Degree 5



END BEHAVIOR: Is the behavior of the graph as x approaches $+\infty$ or $-\infty$

If the degree is EVEN, both ends have the SAME behavior

- If the leading coefficient is positive, both ends are up
- If the leading coefficient is negative, both ends are down

If the degree is odd, the ends have OPPOSITE behavior

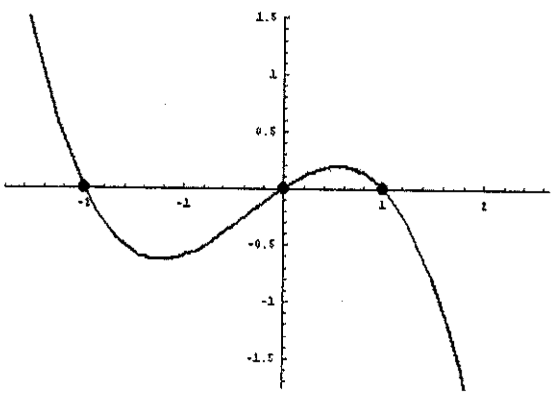
- If the leading coefficient is positive, the right end is up, left down
- If the leading coefficient is negative, the right end is down, left up

Leading Coefficient	Degree	Example	$x \to -\infty$	$x \to \infty$
+	even	$f(x) = x^2$	$f(x) \to$	$f(x) \to$
-	even	$f(x) = -x^2$	$f(x) \to$	$f(x) \to$
+	odd	$f(x) = x^3$	$f(x) \to$	$f(x) \to$
-	odd	$f(x) = -x^3$	$f(x) \to$	$f(x) \to$

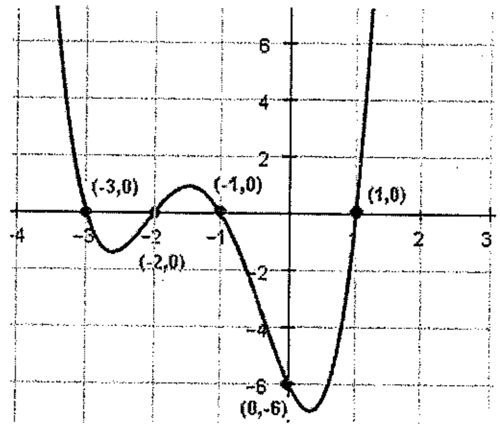
The number "k" is said to be a **zero** of a polynomial if f(k) = 0.

- "k" is often referred to as the **root** or **solution**
- If "k" is a real number, then f(k) = 0 means that the graph crosses the x-axis at that value. "k" can also be referred to as an **x-intercept**

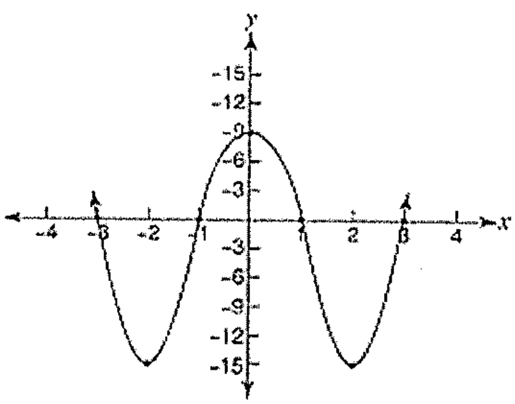
Check out the graphs below and identify any values that represent a zero/solution/root.



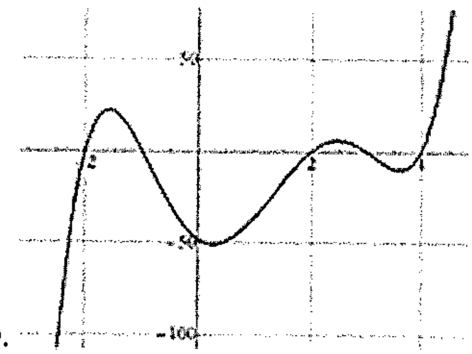
A.
factored equation: ____________________



B.
factored equation: ____________________



C.
factored equation: ____________________



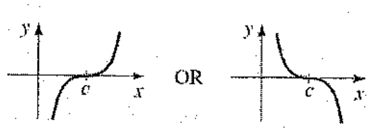
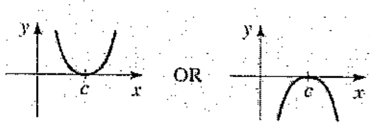
D.
factored equation: ____________________

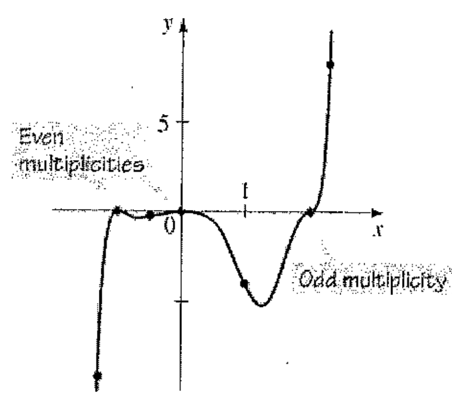
WATCH OUT! Multiplicities of Zeros

If c is a zero of the function P and the corresponding factor (x − c) occurs exactly m times in the factorization of P then we say that c is a zero of **multiplicity** m. One can show that the graph of P crosses the x-axis at c if the multiplicity m is odd and does not cross the x-axis if m is even.

Shape of the Graph Near a Zero of Multiplicity *m*

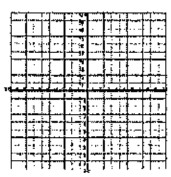
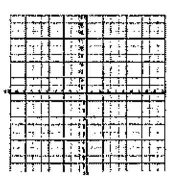
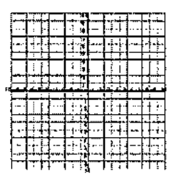
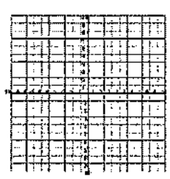
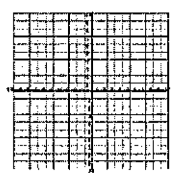
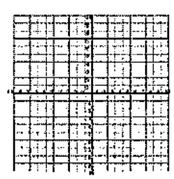
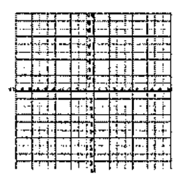
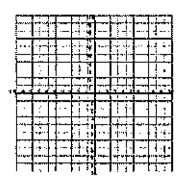
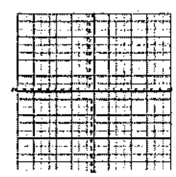
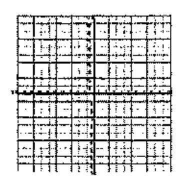
Suppose that *c* is a zero of *P* of multiplicity *m*. Then the shape of the graph of *P* near *c* is as follows.

Multiplicity of *c*	Shape of the graph of *P* near the *x*-intercept *c*
m odd, $m > 1$	
m even, $m > 1$	



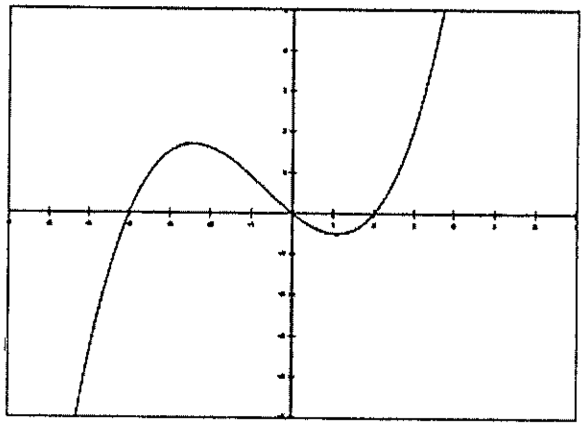
Polynomial Graphs and Zeros

The degree of a polynomial function gives a lot of information…

	$y = ax$ …	$y = ax^2$ …	$y = ax^3$ …	$y = ax^4$ …	$y = ax^5$ …
Type of Polynomial Function	LINEAR	QUADRATIC	CUBIC	QUARTIC	QUINTIC
Domain **Range**					
Maximum number of solutions/zeros (this is equal to the degree of the polynomial)					
Maximum number of turns in the graph (this is one less than the degree of the polynomial)					
Possible shape of the graph *Positive a* *Negative a*	 	 	 	 	 
End behavior *Positive a* *Negative a*					

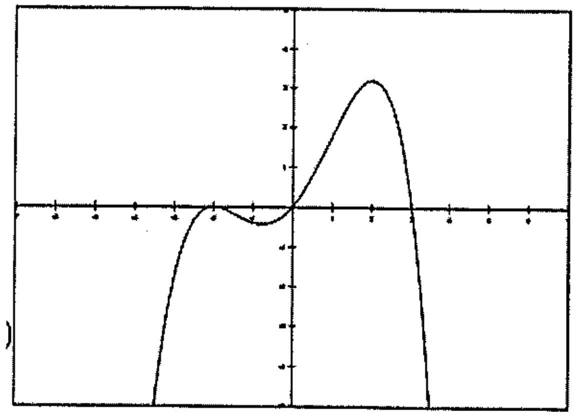
Zeros of a Polynomial Function

Part 1: Look at the graph and state the x-intercepts; watch out for repeated roots!



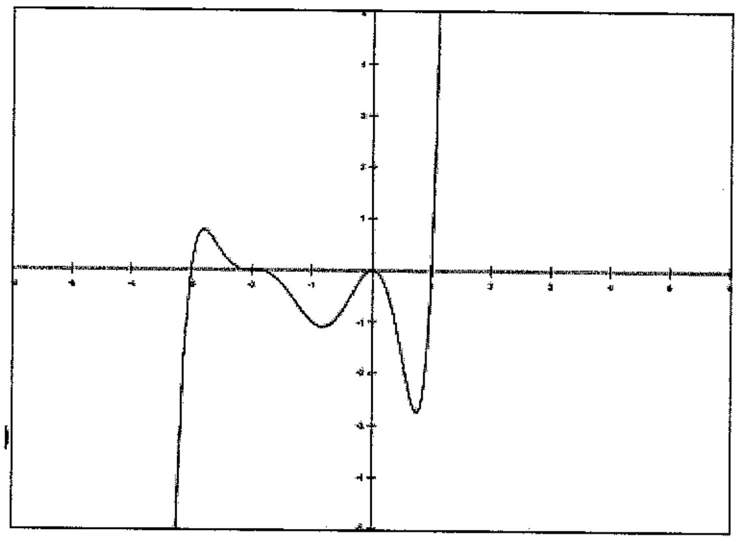
x-intercepts: ____________________

equation: ______________________



x-intercepts: ____________________

equation: ______________________



x-intercepts: ____________________

equation: ______________________

Part 2: Use the calculator to find any exact roots.

A) $f_{(x)} = x^3 - 6x^2 + 11x - 6$

Zeros: ______________

B) $f_{(x)} = x^3 - 9x^2 + 27x - 27$

Zeros: ______________

C) $f_{(x)} = x^3 - 9x^2 + 20x - 12$

Zeros: ______________

Factored Form of each function

A)

B)

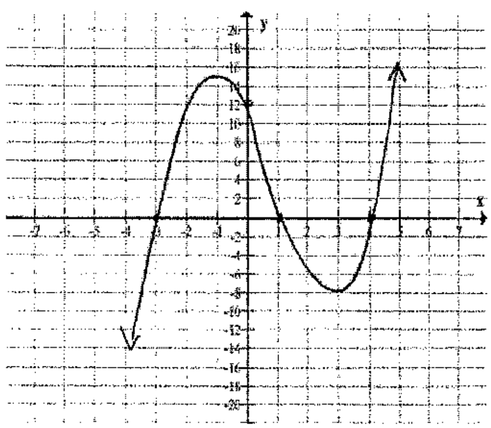
C)

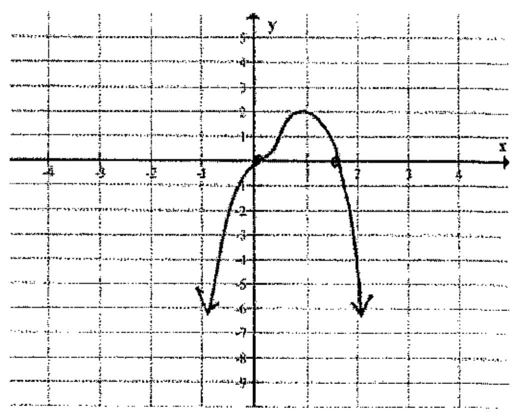
Unit 3: Polynomial Graphs Homework Name ____________________

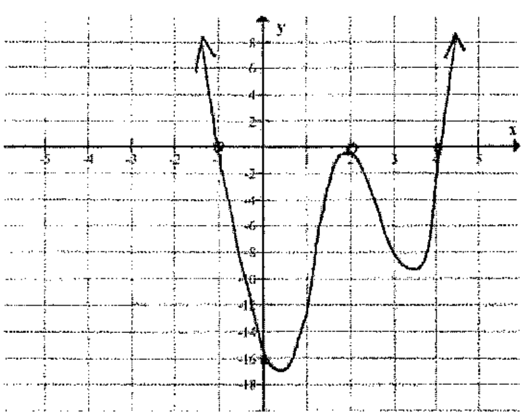
1. Fill in the missing information.

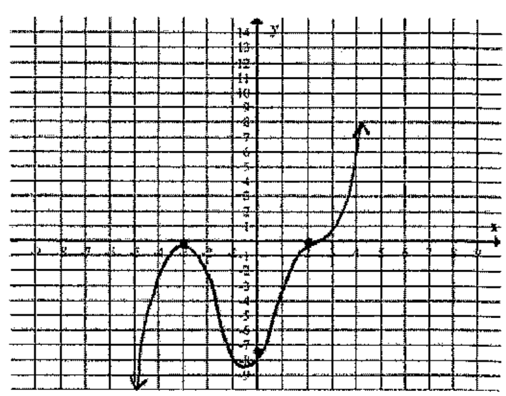
Polynomial Function	Name (degree)	Name (terms)	End Behavior
$f(x) = 3x^2 - 5$			
$y = -x^4 + 6x - 1$			
$g(x) = 6x$			
$h(x) = 5x^2 - 2x^3 + 7x - 3$			

2. Identify the zeros of each function below. Be sure to state any multiplicity.





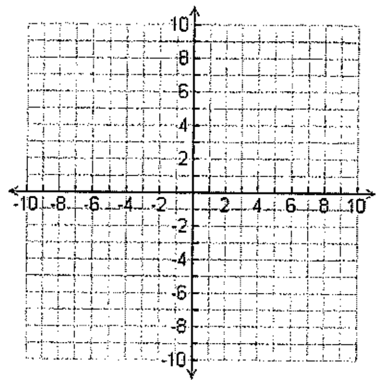




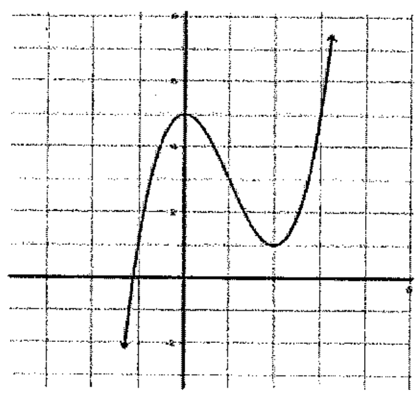
3. Use the given information to complete the missing columns.

Table of Values | Graph | Key features of the function

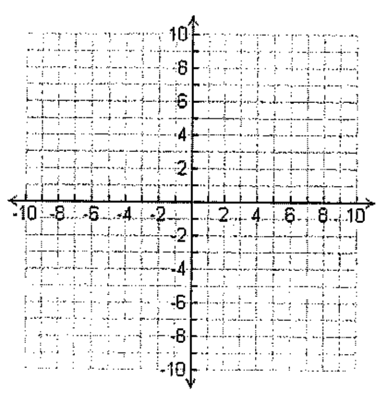
x	y
−3	6
−2	0
−1	−4
0	−6
1	−6
2	−4
3	0
4	6



X	Y

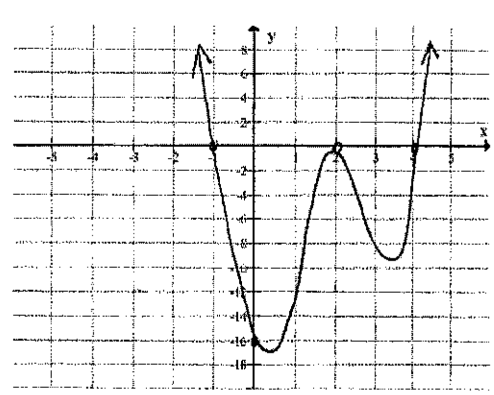


X	Y



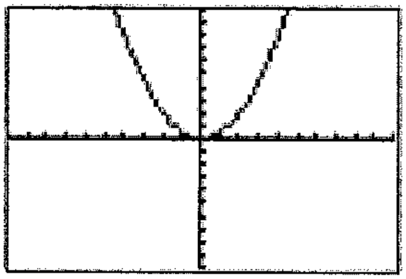
The y-intercept is (0, 7). The zeros are located at x = 4 and x = 7. There is a relative minimum at (-5.5, 1.5) and at (5.5, -2.5). A relative maximum is located at (-1, 8.5). The polynomial is quartic.

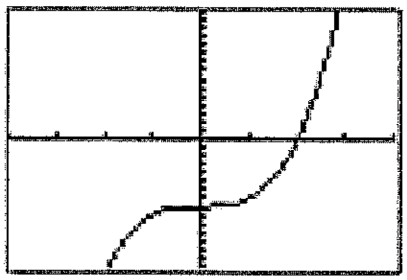
4. Given the graph, state the intervals where the graph is increasing/decreasing and where the graph is positive/negative.

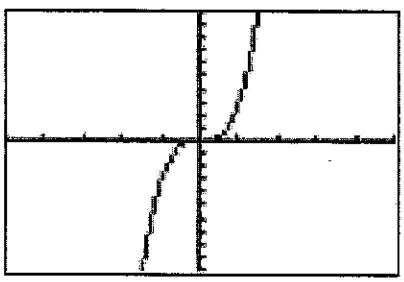


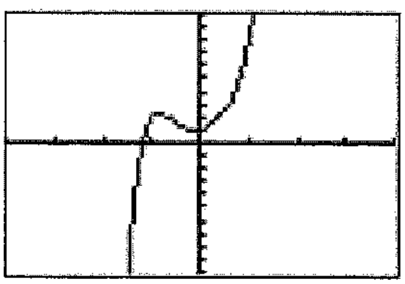
Day 3 Polynomial Matching Warm Up

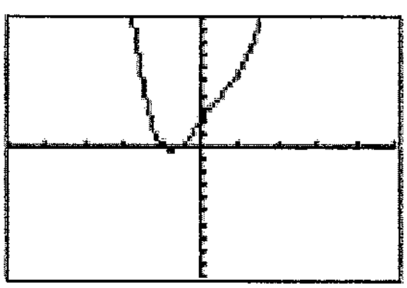
Without using a graphing calculator, match each graph below with the function that it represents.

a. 

b. 

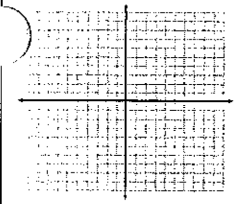
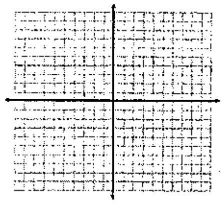
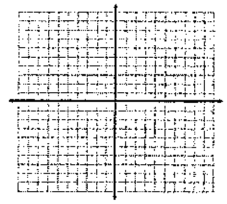
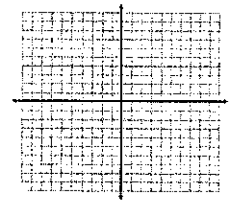
c. 

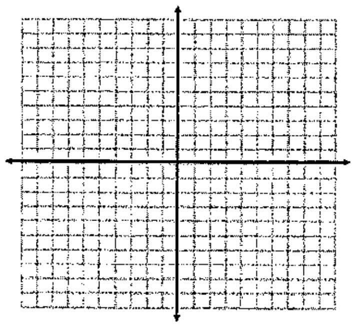
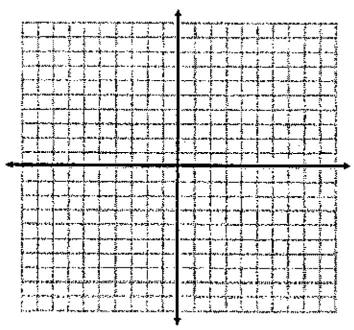
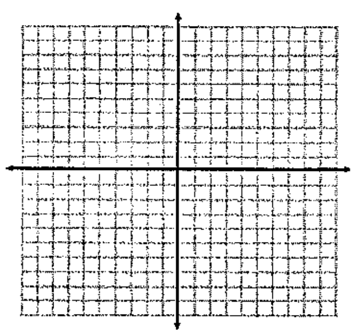
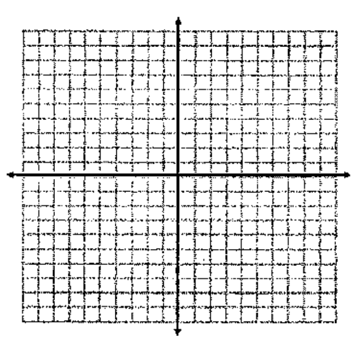
d. 

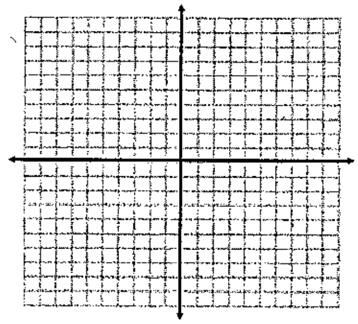
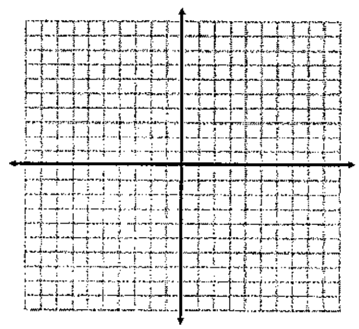
e. 

1. $y = 3x^3$
2. $y = \frac{1}{2}x^2$
3. $y = x^3 - 8$
4. $y = x^4 - x^3 + 4x + 2$
5. $y = 3x^5 - x^3 + 4x + 2$

Math 3 Polynomial Parent Functions

Linear	Quadratic	Cubic	Quartic
			

Function Equation State the type of Function	Sketch the function	Words: The graph moved… (compare to the parent function)
$y = x^3 - 3$ ________________		
$y = (x+5)^2$ ________________		
$y = (\ x+1)^4$ ________________		
$y = -x + 6$ ________________		

Function Equation Parent Name	Graph the function	Words: The graph moved... (compare to the parent function)
$y = (-2x-1)^2 + 4$ ________________		
$y = -(x+1)^3 + 2$ ________________		

General Form of a function $\quad f(x) = a\,(\,x - h) + k$

Summarize the different types of transformations

When a > 1: ________________________________

When 0 < a < 1: ________________________________

When a is negative: ________________________________

When b > 1: ________________________________

When 0 < b < 1: ________________________________

When b is negative: ________________________________

When h is added: ________________________________

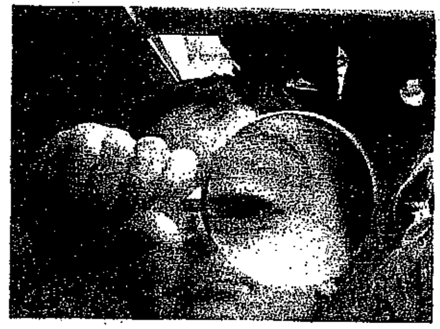
When h is subtracted: ________________________________

When k is added: ________________________________

When k is subtracted: ________________________________

3.8 I Know, What do you know?

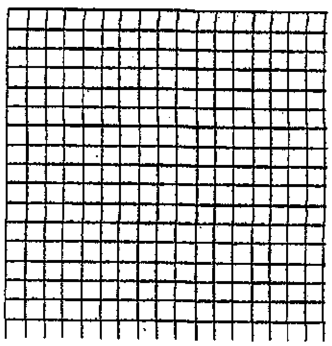
A Practice Understanding Task

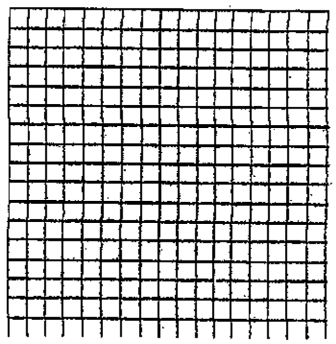


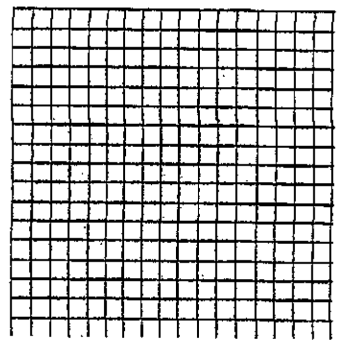
http://www.flickr.com/photos/chrisbrenschmidt/1831955(

Use the information provided to graph and write out the polynomial function in factored form.

	Degree of poly	Given roots (you may have to determine others):	Leading coefficient	Equation (in factored and standard form):
1	3	-2, 1, and 1	-2	
2	4	2i, 4, 0	1	
3	2	$\sqrt{2}$	-1	







If I know... What do you know? For each problem, what I know about a function is given... your job is to complete the table of information with what you know.

4. **Function:**
$f(x) = 2(x-1)(x+3)^2$

End behavior:
$as\ \ x \to -\infty, \quad f(x) \to$ ____
$as\ \ x \to \ \infty, \quad f(x) \to$ ____

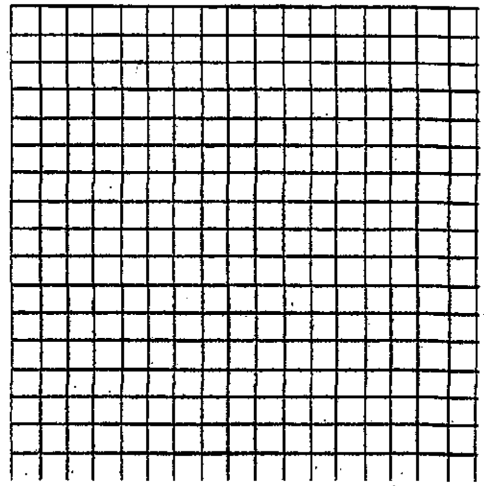
Roots (with multiplicity):

Value of leading co-efficient:

Domain:

Range: All Real numbers

Graph:



5. **Function:**

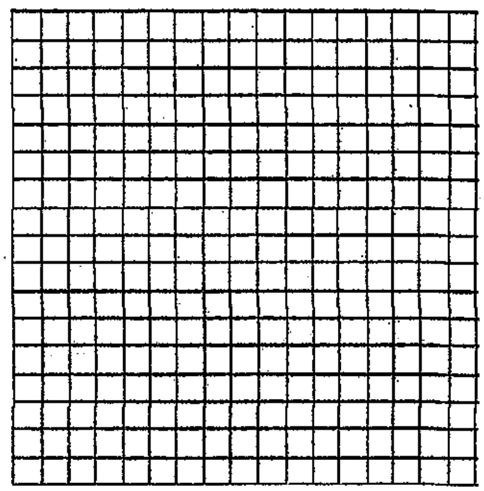
End behavior:
$as\ \ x \to -\infty, \quad f(x) \to \infty$
$as\ \ x \to \ \infty, \quad f(x) \to$ ____

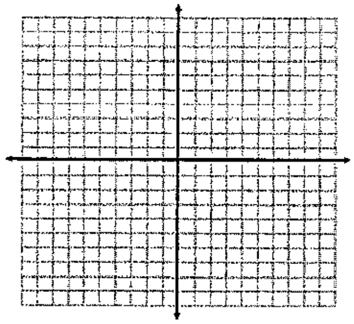
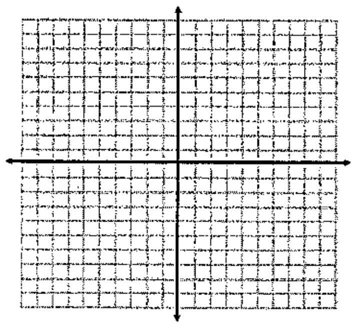
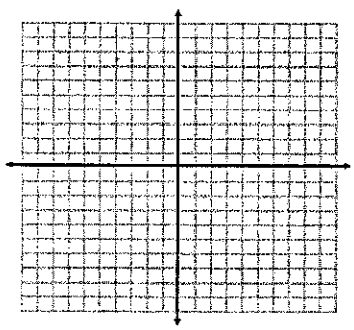
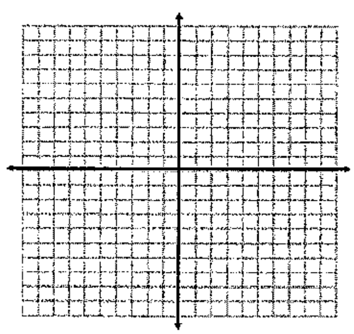
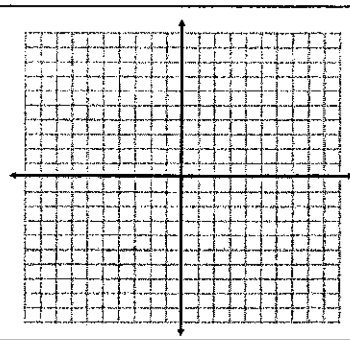
Roots (with multiplicity):
(3,0) m: 1;
(-1,0) m: 2
(0,0) m: 2
Value of leading co-efficient: -1

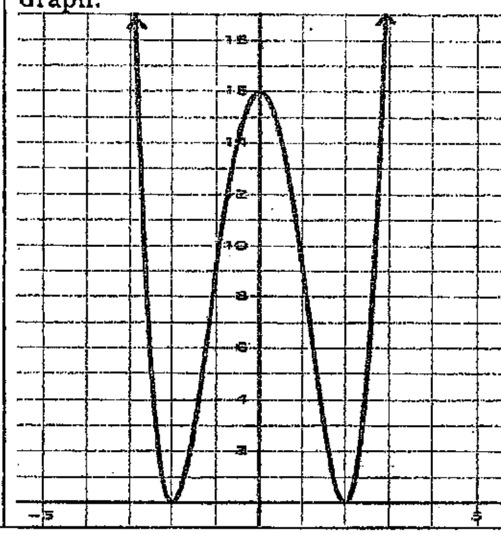
Domain:

Range:

Graph:



Function Equation State the type of Function	Sketch the function	Words: The graph moved… (compare to the parent function)
$y = -(x+4)^3$ ________________		
$y = -x^4 - 2$ ________________		
$y = -\left(\frac{1}{4}x + 1\right)^2 + 3$ ________________		
$y = -2x + 5$ ________________		
$y = -3x^2$ ________________		

6.	Function: **End behavior:** *as* $x \to -\infty$, $f(x) \to$ ____ *as* $x \to \infty$, $f(x) \to$ ____ **Roots (with multiplicity):** **Value of leading co-efficient:** 1 **Domain:** **Range:** **Other:** $f(-2) = 0$	Graph: 

Without using technology, sketch the graph of the polynomial function described. The term "imaginary roots" means complex zeros.

7. A cubic function with a leading coefficient of -2, with two negative zeros and one positive.

8. A quartic function with a leading coefficient of 1, with two negative zeros and one positive double zero.

9. A cubic function with a leading coefficient of -3, with an imaginary root and one positive double root.

10. A quartic function with a leading coefficient of -2, with two negative zeros and one positive double root.

Find all factors and sketch the graph of the polynomial functions.

11. $f(x) = x^3 - x^2$

5. Given the graph below state the following information:

Zeroes: ____________ Degree: ____________ # of turns: ____________

Relative Maximum: ____________ Relative Minimum: ____________

Absolute Maximum: ____________ Absolute Minimum: ____________

End behavior: ________________ ________________

Decreasing Interval(s): ____________ Increasing Interval(s): ____________

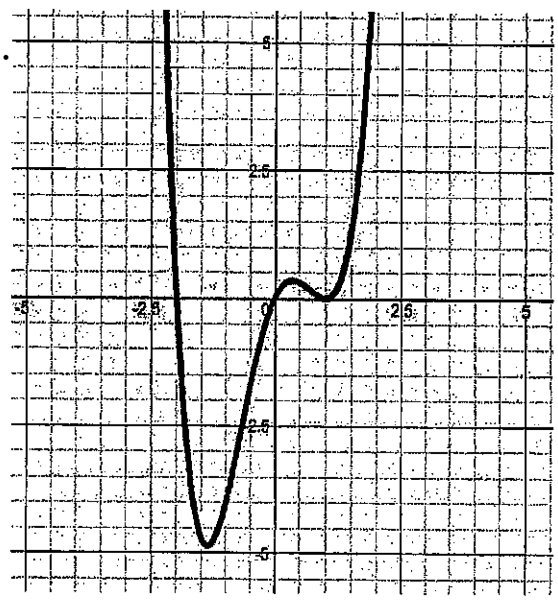
Domain: ______________ Range: ______________

Real zeroes: ____________ Imaginary zeroes: ____________

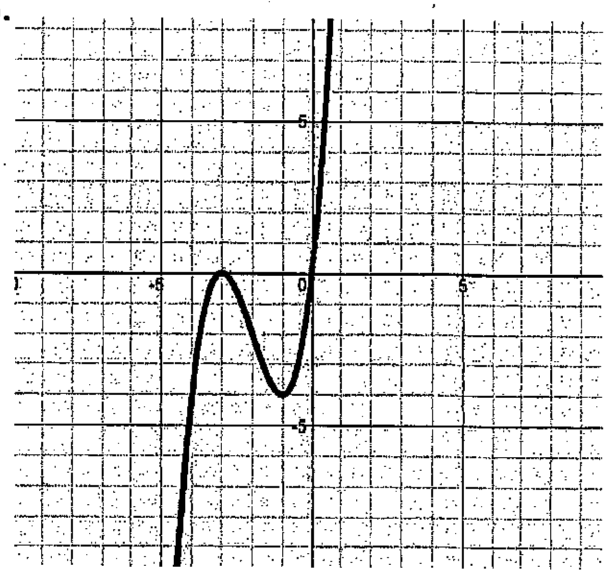
Write the equation in factored form: ____

Write the equation in standard form: ____

5.



6.



6. Given the graph below state the following information:

Zeroes: ____________ Degree: ____________ # of turns: ____________

Relative Maximum: ____________ Relative Minimum: ____________

Absolute Maximum: ____________ Absolute Minimum: ____________

End behavior: ________________ ________________

Decreasing Interval(s): ____________ Increasing Interval(s): ____________

Domain: ______________ Range: ______________

Real zeroes: ____________ Imaginary zeroes: ____________

Write the equation in factored form: ____

Write the equation in standard form: ____

Polynomial Equations and Models

DIAMONDS The weight of an ideal round-cut diamond can be modeled by

$$w = 0.0071d^3 - 0.090d^2 + 0.48d$$

where w is the diamond's weight (in carats) and d is its diameter (in millimeters). According to the model, what is the weight of a diamond with a diameter of 15 millimeters?

CLOTHING The profit P (in millions of dollars) for a T-shirt manufacturer can be modeled by $P = -x^3 + 4x^2 + x$ where x is the number of T-shirts produced (in millions). Currently, the company produces 4 million T-shirts and makes a profit of $4,000,000. What lesser number of T-shirts could the company produce and still make the same profit?

MP3 PLAYERS The profit P (in millions of dollars) for a manufacturer of MP3 players can be modeled by $P = -4x^3 + 12x^2 + 16x$ where x is the number of MP3 players produced (in millions). Currently, the company produces 3 million MP3 players and makes a profit of $48,000,000. What lesser number of MP3 players could the company produce and still make the same profit?

SWIMMING POOL You are designing a rectangular swimming pool that is to be set into the ground. The width of the pool is 5 feet more than the depth, and the length is 35 feet more than the depth. The pool holds 2000 cubic feet of water. What are the dimensions of the pool?

BUSINESS For the 12 years that a grocery store has been open, its annual revenue R (in millions of dollars) can be modeled by the function

$$R = 0.0001(-t^4 + 12t^3 - 77t^2 + 600t + 13{,}650)$$

where t is the number of years since the store opened. In which year(s) was the revenue $1.5 million?

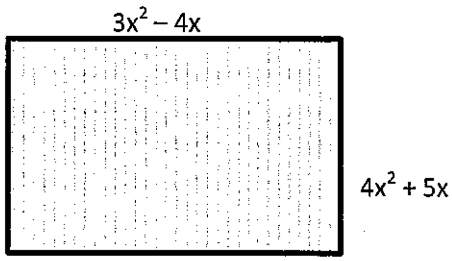
Name:____________________________

Polynomial Applications:

Perimeter, Area, and Volume

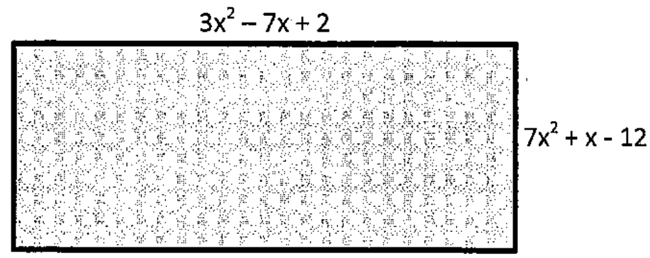
Directions: **Find the PERIMETER of each of the following shapes.**

1.



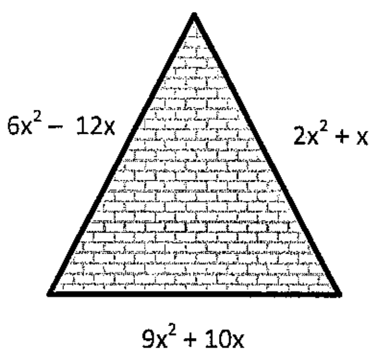
P = ________________

2.



P = ________________

3.



P = ________________

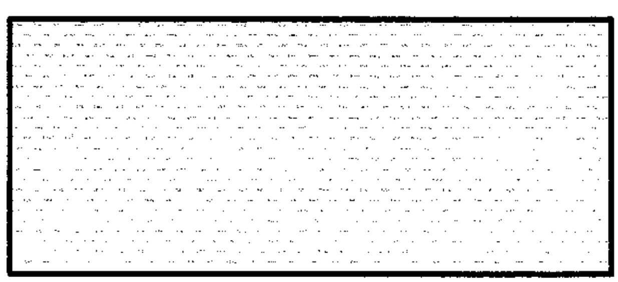
4. The width of a rectangle is $5x - 4$. The perimeter of the rectangle is $14x + 4$. What is the length of the rectangle?

a. $2x - 6$

b. $2x + 6$

c. $9x + 8$

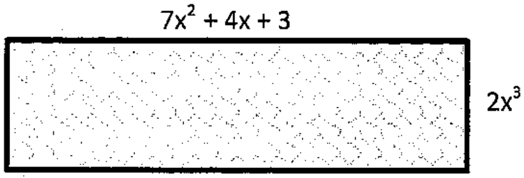
d. $9x - 8$



Name:________________________________

<u>Directions:</u> Find the AREA of each of the following shapes.

5.



A = ________________

6.

A = ________________

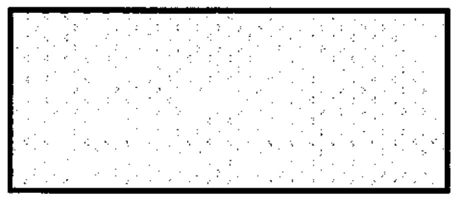
7. Length of a rectangle is $4x^2 + 12x$ and the area of the rectangle is $24x^4 + 72x^3$, what is the width of the rectangle?

a. 6

b. 6x

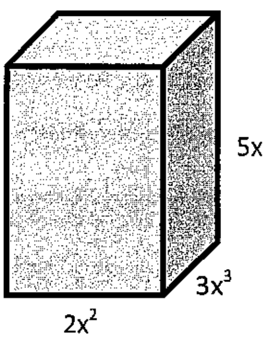
c. $6x^2$

d. $6x^3$



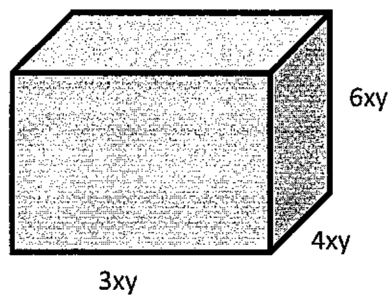
<u>Directions:</u> Find the VOLUME of the following shapes.

8.



V = ________________

9.



V = ________________

10. If the length of a rectangular prism is $4x^2$, the width is $6x^3$, and the volume is $48x^8$, what is the height of the rectangular prism?

a. $2x^3$

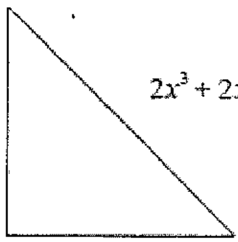
b. $2x^2$

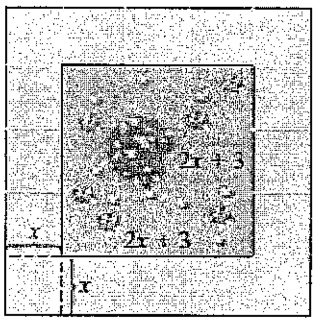
c. 2x

d. 2

GIVEN THE PERIMETER, FIND THE MISSING SIDE.

$P = 2x^3 + 4x^2 + 6x + 3$

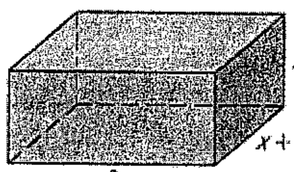




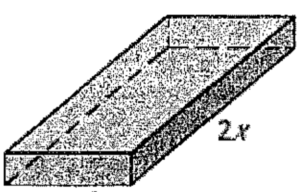
A) FIND THE AREA OF THE GARDEN.

B) FIND THE AREA OF THE WALKWAY AND THE GARDEN.

$V = 2x^3 + 17x^2 + 46x + 40$



Volume = 40



÷POLYNOMIAL÷
÷LONG DIVISION÷

Before you begin long division, always make sure your polynomials are written in standard form. Make sure that any missing terms are written with a 0 coefficient.

Example: $\frac{x^5-5x^3+3x^4-10x^2-30x+50}{x^2+3x-5}$

Example: $x^4 - 6x^2 - 27 \div x + 2$

THE REMAINDER THEOREM & FACTOR THEOREM

How to check whether a linear polynomial is a factor WITHOUT division or a calculator

Is $x + 1$ a factor of $x^3 + 4x^2 + x - 6$?

Is $x - 3$ a factor of $x^3 - 4x^2 + x + 6$?

How to use division to find the other factors when given a zero/solution

$f(x) = 2x^3 + 5x^2 - 22x + 15; 1$

Long Division Practice

Divide each of the polynomials using long division.

1. $(4x^2 - 9) \div (2x + 3)$

2. $(x^2 - 4) \div (x + 4)$

3. $(2x^2 + 5x - 3) \div (x + 3)$

4. $(2x^2 + 5x - 3) \div (x - 3)$

5. $(3x^2 - 13x - 10) \div (x - 5)$

6. $(3x^2 - 13x - 10) \div (x + 5)$

7. $(11x + 20x^2 + 12x^3 + 2) \div (3x + 2)$

8. $(12x^3 + 2 + 11x + 20x^2) \div (2x + 1)$

9. $\frac{x^4 - 1}{x^2 - 1}$

10. $\frac{x^4 - 9}{x^2 + 3}$

POLYNOMIAL DIVISION HOMEWORK

Answer the questions, showing all work, on a separate page

1) Find the quotient and remainder, then write the division statement for each polynomial division.

a) $(x^3+13x^2+39x+20)\div(x+9)$

b) $(x^3-x^2+8x+37)\div(x-2)$

c) $(5x^3+3x^2-5x+3)\div(x-1)$

d) $(-2a^3-11a^2+7a+6)\div(a+6)$

e) $(x^3-12x-20)\div(x+2)$

f) $(x^3-10x-15+7x^2)\div(x+8)$

g) $(4n^3-13n-6)\div(2n+1)$

h) $(x^3+5x^2-2x-24)\div(x^2+7x+12)$

i) $(10a^4-a^3+11a^2+7a+5)\div(5a^2+2a-1)$

j) $(6t^4+4t^3-13t^2-10t-5)\div(2t^2-5)$

2) One factor of $4x^3+15x^2-31x-30$ is $x-2$. Completely factor $4x^3+15x^2-31x-30$.

3) Two factors of $12a^4-39a^2+8a-8a^3+12$ are $a-2$ and $2a+1$. Find the other factors.

4) When $10x^3+mx^2-x+10$ is divided by $5x-3$, the quotient is $2x^2+nx-2$ and the remainder is 4. Find the values for m and n.

ANSWERS

1) a) $x^3+13x^2+39x+20=(x+9)(x^2+4x+3)-7$

b) $x^3-x^2+8x+37=(x-2)(x^2+x+10)+57$

c) $5x^3+3x^2-5x+3=(x-1)(5x^2+8x+3)+6$

d) $-2a^3-11a^2+7a+6=(a+6)(-2a^2+a+1)$

e) $x^3-12x-20=(x+2)(x^2-2x-8)-4$

f) $x^3+7x^2-10x-15=(x+8)(x^2-x-2)+1$

g) $4n^3-13n-6=(2n+1)(2n^2-n-6)$

h) $x^3+5x^2-2x-24=(x^2+7x+12)(x-2)$

i) $10a^4-a^3+11a^2+7a+5$
$=(5a^2+2a-1)(2a^2-a+3)+8$

j) $6t^4-4t^3-13t^2-10t-5$
$=(2t^2-5)(3t^2+2t+1)$

2) $4x^3+15x^2-31x-30=(x-2)(4x+3)(x+5)$

3) other factors are $3a-2$ and $2a+3$

4) $m=-21, n=-3$

EXAMPLES: Using Remainder & Factor Theorems

1) If $P(x) = 3x^5 - x^4 - 5x + 10$, find P(-2) using synthetic substitution.

-2 | 3 -1 0 0 -5 10

P(-2)

Note: Plugging in −2 for x will give the same result!

2) a. If $f(x) = 2x^4 - 8x^2 + 5x - 7$, find f(3)

b. If $p(x) = 6x^3 - 5x^2 + 4x - 17$ find p(3)

c. If $p(x) = 3x^4 + 2x^3 + 4x$ find p(-5)

3) Determine whether $x = 2$ is a zero of $p(x) = 3x^7 - x^4 + 2x^3 - 5x^2 - 4$

For $x = 2$ to be a zero of $p(x)$, p (2) must evaluate to _____.

4) Determine whether $x = -4$ is a solution of $x^6 + 5x^5 + 5x^4 + 5x^3 + 2x^2 - 10x - 8 = 0$

If $x = -4$ is a solution, what must be a factor of the related function? __________

5) Is x − 2 a factor of $f(x) = x^4 - 4x^3 + 5x^2 + 4x - 12$?

2 | 1 -4 5 4 -12

If the remainder = 0, then (x-2) is a factor.

-OR- Find f(2), if it equals zero, then x − 2 is a factor!

6) Factor $f(x) = 2x^3 + 11x^2 + 18x + 9$ given that f(-3) = 0. (means that x + 3 is a factor)

7) Factor $f(x) = 3x^3 + 14x^2 - 28x - 24$ given that x − 2 is a factor.

8) Given that 2 is a zero of $f(x) = x^3 - 2x^2 - 9x + 18$ Find all other zeros.

2	1	-2	-9	18
		2	0	-18
	1	0	-9	0

$\Rightarrow$ $x^2 - 9 = 0$

$x^2 = 9$

$x = 3, -3$

9) Solve given that $-\frac{1}{2}$ is a root of $f(m) = 2m^3 - 5m^2 - 13m - 5$.

10) Solve $p(x) = x^3 + x + 10$, if -2 is a root.

(HONORS) More finding zeros:

<u>**HINT:**</u> Use the sum/product rule to write a quadratic factor, then use long division to divide the given equation by the quadratic factor you found. Set the quotient = 0 and find the remaining roots.

11.) Solve $x^4 - 6x^3 + 6x^2 + 24x - 40 = 0$ given $3 + i$ is a root.

12.) Solve $x^4 + x^3 + 6x^2 - 14x - 20 = 0$ if $-1 + 3i$ is a root.

Polynomial Division Worksheet

Divide using Synthetic Division

1. $(3y^3 + 2y^2 - 32y + 2) / (y - 3)$

2. $(2b^3 + b^2 - 2b + 3) / (b + 1)$

3. $(2c^3 - 3c^2 + 3c - 4) / (c - 2)$

4. $(3x^3 - 2x^2 + 2x - 1) / (x - 1)$

5. $(t^4 - 2t^3 + t^2 - 3t + 2) / (t - 2)$

6. $(3r^4 - 6r^3 - 2r^2 + r - 6) / (r + 1)$

7. $(z^4 - 3z^3 - z^2 - 11z - 4) / (z - 4)$

8. $(2b^3 - 11b^2 + 12b + 9) / (b - 3)$

9. $(6s^3 - 19s^2 + s + 6) / (s - 3)$

10. $(x^3 + 2x^2 - 5x - 6) / (x - 2)$

11. $(x^3 + 3x^2 - 7x + 1) / (x - 1)$

12. $(n^4 - 8n^3 + 54n + 105) / (n - 5)$

13. $(2x^4 - 5x^3 + 2x - 3) / (x - 1)$

14. $(z^5 - 6z^3 + 4x^2 - 3) / (z - 2)$

15. $(y^4 + 3y^3 + y - 1) / (y + 3)$

Divide using long division:

16. $(4s^4 - 5s^2 + 2s + 3) / (2s - 1)$

17. $(2x^3 - 3x^2 - 8x + 4) / (2x + 1)$

18. $(4x^4 - 5x^2 - 8x - 10) / (2x - 3)$

19. $(6j^3 - 28j^2 + 19j + 3) / (3j - 2)$

20. $(y^5 - 3y^2 - 20) / (y - 2)$

Dividing Polynomials - EXAMPLES

Dividing by a monomial

1. $(-30x^3y + 12x^2y^2 - 18x^2y) \div (-6x^2y)$

Divide using Long Division

2. $(6x^2 - x - 7) \div (3x + 1)$

3. $(4x^2 - 2x + 6)(2x - 3)^{-1}$

4. $(4x^3 - 8x^2 + 3x - 8) \div (2x - 1)$

5. $(2x^3 - 3x^2 - 18x - 8) \div (x - 4)$

6. $(2x^4 + 3x^3 + 5x - 1) \div (x^2 - 2x + 2)$

Divide using Synthetic Division

7. $(2x^2 + 3x - 4) \div (x - 2)$

8. $(x^4 - 3x^3 + 5x - 6) \div (x + 2)$

9. $(2x^3 + 4x - 6) \div (x + 3)$

10. $(x^4 - 2x^3 + 6x^2 - 8x + 10) \div (x + 2)$

(HONORS) 11. $(6x^4 - x^3 + 3x + 5) / (2x + 1)$

Zeros & Remainder Theorem, Factor Theorem Worksheet

#1-6 Find the zeros of the polynomials and state the multiplicity of each zero:

1. $f(x) = (x + 4)^3 (3x - 4)$
2. $f(x) = 2x^5 - 8x^4 - 10x^3$
3. $f(x) = (9x^2 - 25)^4 (x^2 + 16)$
4. $f(x) = (x^2 + x - 2)^2 (x^2 - 4)$
5. $f(x) = x(x + 2)^3 (x - 5)$
6. $f(x) = 2x(x+3)^2(4x - 1)$

#7&8 Write a polynomial equation in standard from having the given roots:

7. 2, 3,- 1 — (7A) 5, 2i, -2i
8. 1 multiplicity 2, 0 — (8A) -3, 2+i, __
9. -2 multiplicity 2, 1, 2 — (9A) $\sqrt{3}$, __, -2

#10-15 Use synthetic division to show that c is a zero of f(x).

10. $f(x) = 3x^4 + 8x^3 - 2x^2 - 10x + 4$; $c = -2$
11. $f(x) = 4x^3 - 9x^2 - 8x - 3$; $c = 3$
12. $f(x) = 2x^3 + 5x^2 - 4x - 3$; $c = 1$
13. $f(x) = 2x^4 + x^3 - 14x^2 + 5x + 6$; $c = -3$
14. $f(x) = 4x^3 - 6x^2 + 8x - 3$; $c = ½$
15. $f(x) = 27x^4 - 9x^3 + 3x^2 + 6x + 1$; $c = -1/3$

16. Factor $f(x) = 9x^3 + 6x^2 - 3x$ if you know $(x+1)$ is a factor.

17. Factor $f(x) = x^3 - 2x^2 - 9x + 18$ if you know $(x+3)$ is a factor.

18. Factor $y = x^3 - 4x^2 - 3x + 18$ if you know that $(x+2)$ is a factor.

19. Show that -3 is a zero of multiplicity 2 of the polynomial function

$P(x) = x^4 + 7x^3 + 13x^2 - 3x - 18$ and express P (x) as a product of linear factors.

20. Show that -1 is a zero of multiplicity 4 of the polynomial function

$f(x) = x^5 + x^4 - 6x^3 - 14x^2 - 11x - 3$ and express f (x) as a product of linear factors.

21. Find a polynomial function of degree 4 such that both -2 and 3 are zeros of multiplicity 2.

22. Find a polynomial function of degree 5 such that -2 is a zero of multiplicity 3 and 4 is a zero of multiplicity 2.

23. Determine k so that that $f(x) = x^3 + kx^2 - kx + 10$ is divisible by $x + 3$.

24. Find k so that when $x^3 - x^2 - kx + 10$ is divided $x - 3$, the remainder is -2.

25. Find k so that when $x^3 - kx^2 - kx + 1$ is divided by x-2, the remainder =0

26. Determine k so that that $f(x) = 2kx^3 + 2kx - 10$ is divisible by $x - 2$.

27. SOLVE $x^3 + 4x^2 - 5x = 0$ completely.

#28-35 HONORS

28. SOLVE $x^4 + 7x^2 - 18 = 0$ completely.

29. Determine all values of k so that $f(x) = k^2x^2 - 4kx + 3$ is divisible by $x - 1$.

39. Find the remainder if the polynomial $3x^{100} + 5x^{85} - 4x^{38} + 2x^{17} - 6$ is divided by x+1

31. Write a cubic equation having zeros 2, $\frac{3}{4}$ and -1.

32. Write the quartic equation having zeros $2i$ and $3 - i$.

33. Write the cubic equation having zeros $\frac{-2}{3}$ and $2 + 3i$

34. SOLVE $2x^4 - 17x^3 + 47x^2 - 32x - 30 = 0$ given that $3+i$ is a root.

35. SOLVE $x^4 - x^3 + x^2 + 9x - 10 = 0$ knowing $1 - 2i$ is a root.

Rational Root Theorem

HONORS

EXAMPLES: Find the possible rational roots, then find all the zeros.

1. $3x^3 - x^2 - 15x + 5 = 0$

2. $x^4 - 5x^3 + 9x^2 - 7x + 2 = 0$

PRACTICE:

1. **Solve** $2x^4 + 3x^3 - 11x^2 + 2x + 4 = 0$
2. **Solve** $x^4 + 5x^2 - 6 = 0$
3. **Find all the zeros of** $f(x) = x^4 - x^3 + 2x^2 - 4x - 8$
4. **Find all the roots of** $f(x) = 2x^4 - x^3 - 8x^2 + x + 6$
5. **Find all the zeros of** $f(x) = x^3 + 3x^2 + 3x + 2$
6. **Find all the solutions of** $0 = 15x^4 + 68x^3 - 7x^2 + 24x - 4$

Precalculus Name____________________ ID: 1

Assignment

Date________________ Period____

Find all zeros. Show your work, exact answers in simplest form

1) $f(x) = x^6 - 64$

2) $f(x) = x^4 - 2x^2 - 35$

3) $f(x) = x^4 - 6x^2 + 8$

4) $f(x) = x^5 + 5x^3 - 24x$

5) $f(x) = x^3 - 2x^2 - 4x + 8$

6) $f(x) = x^3 - 4x^2 + 3x$

7) $f(x) = x^4 - 18x^2 + 81$

8) $f(x) = x^5 + 4x^3 + 3x$

9) $f(x) = x^6 - 64$

10) $f(x) = x^6 - 16x^3 + 64$

Day 7 Polynomial Division Warm Up

1. Determine if $(x-1)$ a factor of $3x^3-4x^2+x+2$. Explain how you know.

2. Find all Roots/Zeros of $x^3+6x^2+11x+6$.
(Hint: Use the calculator to locate a zero)

3. Solve by finding all Roots/Zeros of $2x^4+7x^3+4x^2-7x-6=0$.
(Hint: Use a calculator to locate a zero)

3.3 All About Behavior

A Practice Understanding Task

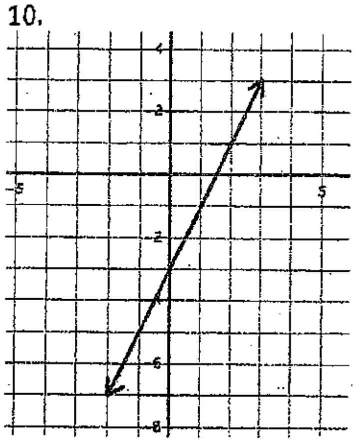
Part I: For each situation:

- Determine the function type. If it is a polynomial, also state the degree of the polynomial and whether it is an even degree polynomial or an odd degree polynomial.
- For each, state the end behavior based on your knowledge of the function
  Use the format: $As\ x \to -\infty, f(x) \to$ _____ $and \quad as\ x \to \infty\ f(x) \to$ _____

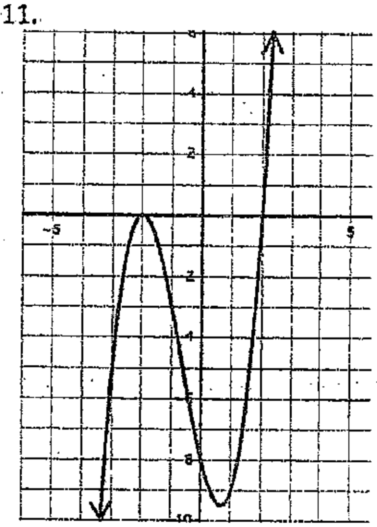
1. $f(x) = 3 + 2x$
2. $f(x) = x^4 - 16$
3. $f(x) = 3^x$
4. $f(x) = x^3 + 2x^2 - x + 5$
5. $f(x) = -2x^3 + 2x^2 - x + 5$
6. $f(x) = log_2 x$
7. $f(x) = -2(x-3)(x+4)$
8. $f(x) = \sqrt{x} - 3$
9. $f(x) = 3(x-1)(x+2)(x-4)$

Use the graphs below to describe the end behavior of each function. Use the same format as above.

10.



11.



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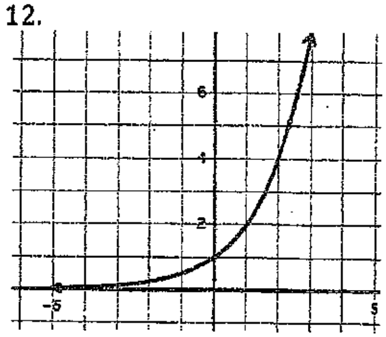

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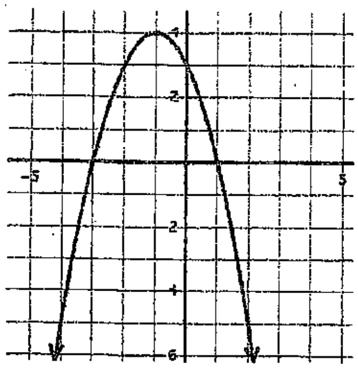
13.



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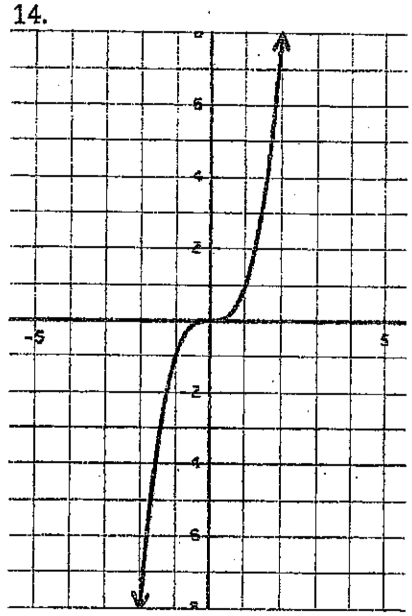

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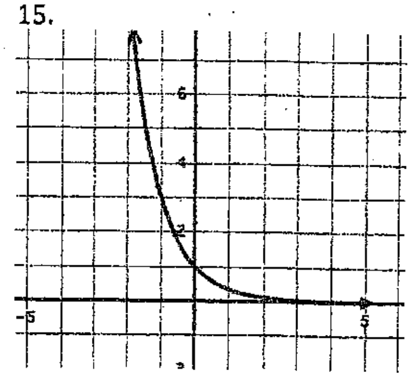

Part II: Use the functions from problems 1-18 to answer the following without finding the solution to each problem. Write a short explanation for each answer.

19. Compare problems 4 and 5: Which has the greatest value as $x \to \infty$?
20. Compare problems 6 and 12: Which has the greatest value as $x \to \infty$?
21. Compare problems 8 and 10: Which has the greatest value at as $x \to \infty$?
22. Compare problems 2 and 4: Which of these two polynomials has the highest degree?
23. Compare problems 7 and 13: Which has the highest maximum value?
24. Compare problems 8 and 11: Which has the greatest average rate of change from [15, 20]?
25. Compare problems 12 and 14: Which grows faster as $x \to \infty$?
26. Extension: Create three comparison problems of your own (be sure you know the answer).

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Name Polynomial Functions | 3.3

Set

Topic: Determine the function type and state the end behavior.

11. $f(x) = x^2 + 12x - 1$

12. $g(x) = 4 * 2^x$

13. $h(x) = -x^3 + 1$

14. $p(x) = -x^2 + 3x - 1$

Use the equations above to answer the following:

15. Which function above has the greatest value at $x = 1{,}000$?

16. Which function above is *always* increasing?

17. Which function above is *always* decreasing?

18. Which function above has a relative maximum value?

19. Which function above has a relative minimum value?

Go

Topic: Solve for x.

20. $x^2 - 16 = 0$

21. $x^2 + 4x + 3 = 0$

22. $x^2 - 5x + 6 = 0$

23. $x^2 + 4x = 12$

24. $(x + 4)(x - 3)(x + 1) = 0$

25. $x(x^2 - 6x + 9) = 0$

Topic: Multiply.

26. $(x - 7)(x + 7)$

27. $(3x - 5)(3x + 5)$

28. $(x - 9)(x - 9)$

29. $(x + 1)(x + 1)$

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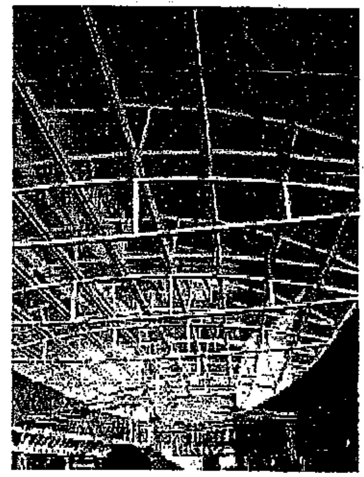
Name



http://www.flickr.com/photos/nguyendai/479560639

Ready, Set, Go!

Ready

Topic: Factoring Special Products

1. $4x^2 - 25$

2. $9x^2 - 16y^2$

3. $a^2x^2 - b^2$

4. $64x^3 - 125$

5. $27x^3 + 8$

6. $1000x^3 - y^3$

Set

Topic: Find all zeros of each polynomial, then sketch the graph. Use technology to check your answer.

7. $f(x) = x^2 - 25$

8. $g(x) = 4x^2 - 9$

9. $h(x) = x(x^2 - 5x + 6)$

Go

Topic: Multiply polynomials

10. $(x-3)(x+3)$

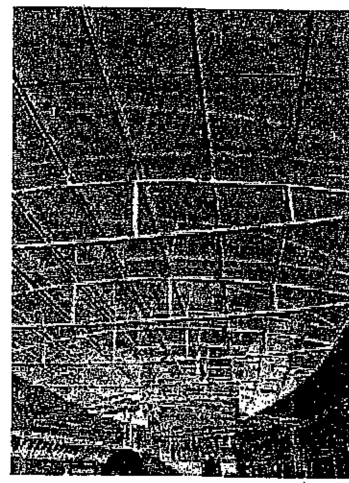
11. $(x+4)(x+4)$

12. $(x-2)(x^2+2x+4)$

13. $(x+1)(x^2-x+1)$

3.6 Seeing Structure

A Solidify Understanding Task



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Claire and Carmella were having a discussion about how easy it is to graph polynomial functions. Claire stated: "All you need to know to sketch the graph of a polynomial function is the degree of the polynomial. The degree will tell you the end behavior and the number of times the graph will cross the x-axis." Carmella mostly agreed, however, thought there was something not quite right with this statement.

1. Modify Claire's statement about sketching the graph of a polynomial function:

For each function, identify the end behavior and roots (including the multiplicity) of the function.

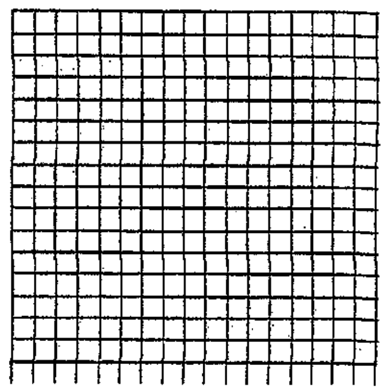
2. Equation: $f(x) = -x(x-2)(x-4)$

Intercepts:

End behavior:

$as \ \ x \to -\infty, \quad f(x) \to$ ____
$as \ \ x \to \infty, \quad f(x) \to$ ____

Graph:



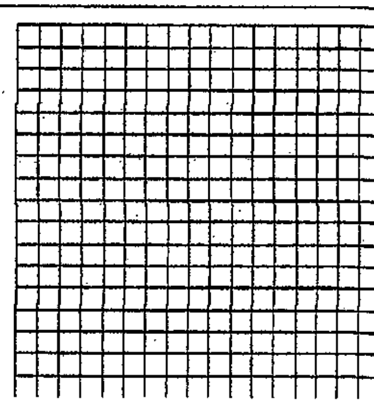
3. Equation: $f(x) = x(x^2 + 4x + 4)$

Intercepts:

End behavior:

$as \ \ x \to -\infty, \quad f(x) \to$ ____
$as \ \ x \to \infty, \quad f(x) \to$ ____

Graph:



4. Equation: $f(x) = g(x) = x^3 - x^2$

Graph:

Intercepts:

End behavior:

$as\ \ x \to -\infty, \qquad f(x) \to$ ____
$as\ \ x \to \ \ \infty, \qquad f(x) \to$ ____

5. Equation: $f(x) = x^4 - 16$

Graph:

Intercepts:

End behavior:

$as\ \ x \to -\infty, \qquad f(x) \to$ ____
$as\ \ x \to \ \ \infty, \qquad f(x) \to$ ____

6. Equation: $f(x) = x^3 - 2x^2 - 3x$

Graph:

Intercepts:

End behavior:

$as\ \ x \to -\infty, \qquad f(x) \to$ ____
$as\ \ x \to \ \ \infty, \qquad f(x) \to$ ____

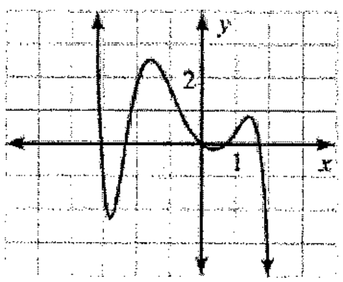
7. Explain how you are able to graph a polynomial that is not already in factored form?

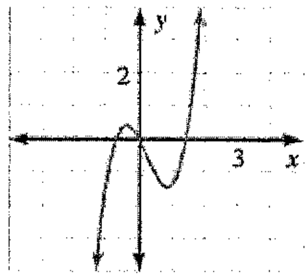
8. If you know one root of a cubic function, can you find the others? Explain?

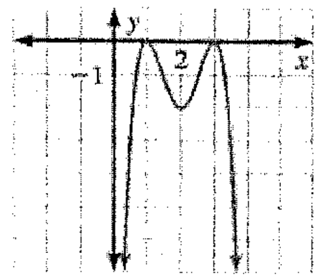
Unit 3 Review

1. Look at the graphs below and answer the following:

   a) What is the degree?
   b) How many zeros does the function have?
   c) Describe the end behavior
   d) State the interval(s) where the function is increasing
   e) Circle any extrema







2. Which polynomial function has zeros at 5, –4, and –3 ?

a. $f(x) = x^3 - 60x^2 + 2x - 23$

b. $f(x) = x^3 + 2x^2 - 23x + 7$

c. $f(x) = x^3 - 17x^2 - 420x + 7$

d. $f(x) = x^3 + 2x^2 - 23x - 60$

3. Find the zeros of $f(x) = (x + 2)^6(x + 3)^4$ and state the multiplicity.

a. –2, multiplicity 6; 4, multiplicity –3

b. –2, multiplicity 6; –3, multiplicity 4

c. 6, multiplicity –2; –3, multiplicity 4

d. 6, multiplicity –2; 4, multiplicity –3

4. Divide $-x^3 + 4x^2 - x - 3$ by $x + 2$.

a. $-x^2 + 6x - 13$

b. $-x^2 + 2x + 11$, R –29

c. $-x^2 + 2x + 11$

d. $-x^2 + 6x - 13$, R 23

5. Divide $(x^4 + 12x^3 - 91x^2 + 26x + 20) \div (x - 5)$

a. $x^3 + 17x^2 - 6x - 4$

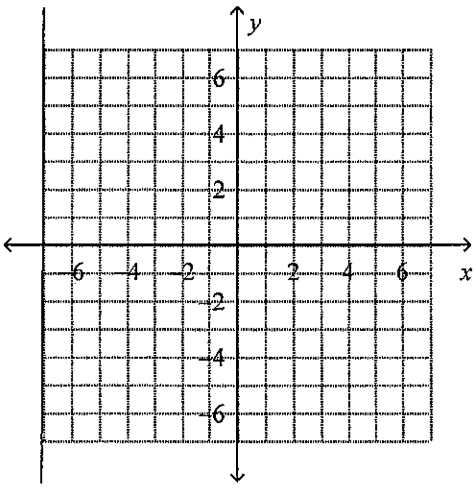
b. $x^3 - 22x^2 - 79x + 34$

c. $x^3 + 12x^2 - 22x + 34$

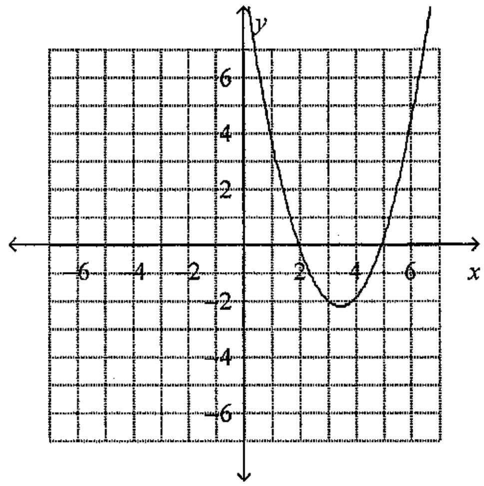
d. $x^3 - 6x^2 - 4x + 17$

6. Find the zeros of $y = x(x-5)(x-2)$. Then graph the equation.

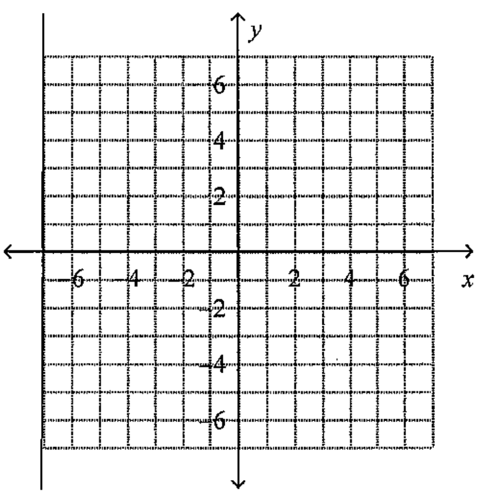
a. 5, 2, –5



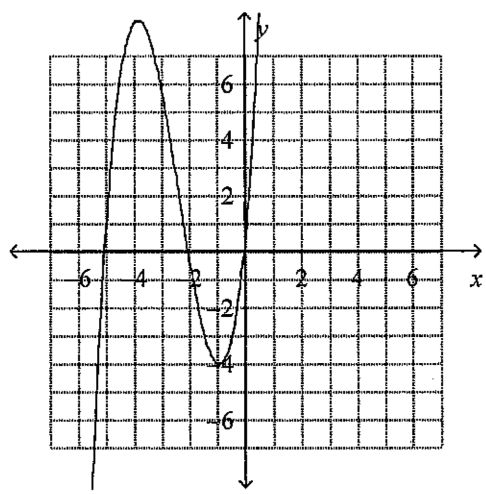
c. 5, 2



b. 0, 5, 2



d. 0, –5, –2



7. Determine which binomial is a factor of $-2x^3 + 14x^2 - 24x + 20$.

a. $x + 5$ b. $x + 20$ c. $x - 24$ d. $x - 5$

Find the roots of the polynomial equation

8. $x^3 - 2x^2 + 10x + 136 = 0$

a. $-3 \pm 5i, -4$

c. $-3 \pm i, 4$

b. $3 \pm 5i, -4$

d. $3 \pm i, 4$

9. $2x^3 + 2x^2 - 19x + 20 = 0$

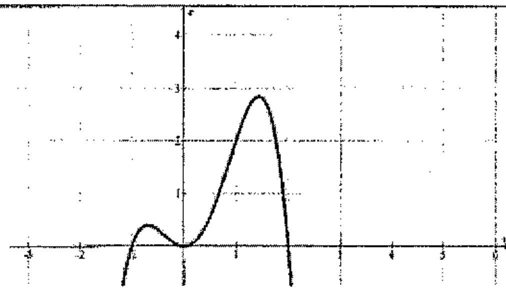
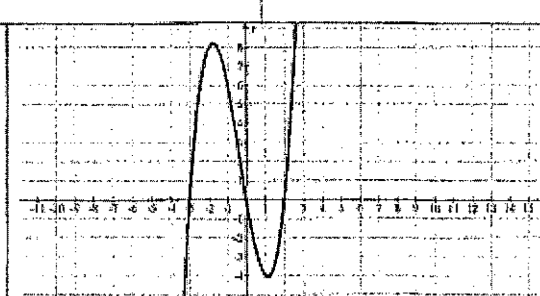
a. $\frac{3+i}{2}, \frac{3-i}{2}, -4$

b. $\frac{-3+2i}{2}, \frac{-3-2i}{2}, 4$

c. $\frac{-3+i}{2}, \frac{-3-i}{2}, -4$

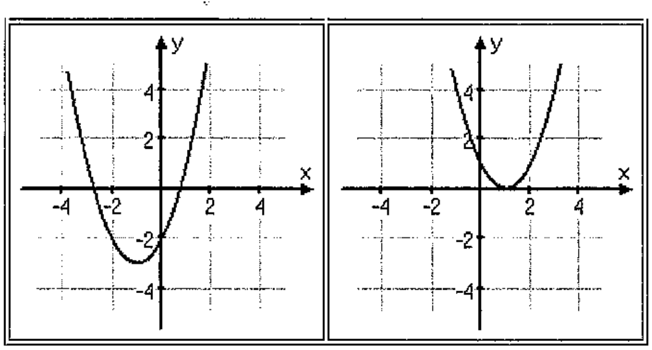
d. $\frac{3+2i}{2}, \frac{3-2i}{2}, 4$

10. Complete the following table

Convert factors to roots	(x + 5)	(x – 3)	(2x + 8)
Convert the roots to factors	X = 7	X = -9	X = 1/3
Identify the FACTORS of the roots shown in the graph	 Factors:	 Factors:	
Multiplicity of the functions graphed above	Root x = 0, multiplicity = ________ Root x = -1, multiplicity = ________ Root x = 2, multiplicity = ________	Root x = -3, multiplicity = ________ Root x = -1, multiplicity = ________ Root x = 2, multiplicity = ________	
Multiplicity of the each root in the function	$(x-3)^2(x+1)(x-2)^3$ Root: x = 3, multiplicity = _____ X = -1, multiplicity = _____ X = 2, multiplicity = ______	$(x-4)(x)(x+3)^5$ Root: x = 4, multiplicity = _____ X = 0, multiplicity = _____ X = -3, multiplicity = ______	

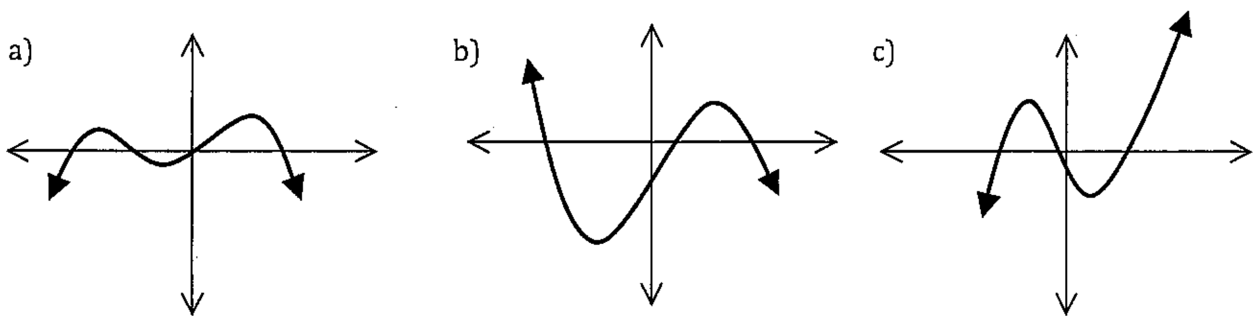
11. Write an equation for the transformation of x^3 three units left, two units up and reflected across the x-axis.

12. Write an equation for each graph below as a transformation from $y = x^2$



Polynomial Review #2

1. Is -3 a zero of $p(x) = 2x^4 + 9x^3 - 7x + 10$? Why or why not?
2. Is $(x + 7)$ a factor of $p(x) = x^4 + 9x^3 + 15x^2 + 5x - 14$? Why or why not?
3. Find $p(3)$ for $p(x) = 3x^4 - 11x^3 - x^2 + 15x - 12$?
4. Factor $p(x) = 3x^3 + 14x^2 - 7x - 10$ completely, given $p(-5) = 0$
5. Write the polynomial in factored form with zeros: 1 multiplicity 3, 0, -4 ?
6. Solve $p(x) = x^3 - 3x^2 - 11x - 7$ given that -1 is a zero.
7. Factor $p(x) = 6x^3 - 23x^2 - 6x + 8$ if $(x - 4)$ is a factor.
8. Solve #7.
9. Sketch the graph of $p(x) = -1(x - 2)(x + 3)(x + 1)$ (no calc)
10. Solve $p(x) = x^3 - 11x^2 + 36x - 36$ if $(x - 6)$ is a factor.
11. Solve $p(x) = 15x^3 - 119x^2 - 10x + 16$ if 8 is a zero.
12. Divide $x^4 - 3x^3 + 18x^2 - 12x + 16$ by $x - 3$ using long division.
13. One root of $2x^3 - 10x^2 + 9x - 4 = 0$ is 4. Find the other roots.
14. If $3 + 2i$ is a zero of a polynomial, what has to be another zero?
15. Describe the end behavior of each: (a) $f(x) = x^5 - x^3 - x^2 + x + 2$; (b) $h(x) = -x^4 - 9x^2$
16. Approximate to the nearest tenth the real zeros of $f(x) = x^3 - 6x^2 + 8x - 2$. (Use a calculator)
17. For $y = x(x + 3)(x - 1)^2$, determine the zeros and their multiplicity.
18. Write a polynomial function with zeros 1 and 2 (of multiplicity 3) in standard form.
19. Use synthetic division to find $f(-2)$ if $f(x) = 4x^5 + 10x^4 - 11x^3 - 22x^2 + 20x + 10$.
20. Factor: $2x^3 + 15x^2 - 14x - 48$ if $(x - 2)$ is a factor.
21. Determine if the degree of the functions below is even or odd. How many real zeros does each have?



(Honors) 22. Find a third degree polynomial with zeros -4 and $2 - 3i$.

(Honors) 23. Write a cubic equation in **standard form** having zeros 3 and $2 + i$.

(Honors) 24. Find a polynomial equation having roots -2 and $3 + i$.

(Honors) 25. Find all zeros for $p(x) = 2x^4 + 3x^3 + 6x^2 + 12x - 8$ if $2i$ is a zero. (no calc)

(Honors) 26. Find all the <u>POSSIBLE</u> rational roots of $p(x) = 3x^4 + 10x^3 - 8x^2 + x - 15$

(Honors) 26. Find all the roots for: $f(x) = 3x^4 + 14x^3 + 14x^2 - 8x - 8$ (no calc)

(Honors) 27. Find all the roots for $p(x) = 3x^3 - x^2 - 6x + 2$ (no calc)

3. Given the graph below state the following information:

Zeroes: ____________ Degree: ____________ # of turns: ____________

Relative Maximum: ____________ Relative Minimum: ____________

Absolute Maximum: ____________ Absolute Minimum: ____________

End behavior: ________________ ________________

Decreasing Interval(s): ____________ Increasing Interval(s): ____________

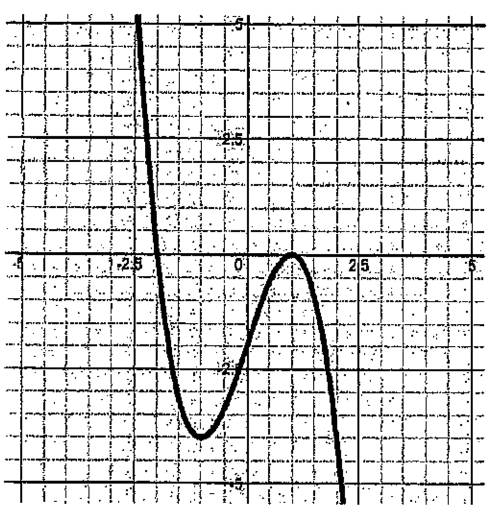
Domain: ______________ Range: ______________

Real zeroes: ____________ Imaginary zeroes: ____________

Write the equation in factored form: ______________________________________

Write the equation in standard form: ______________________________________

3.



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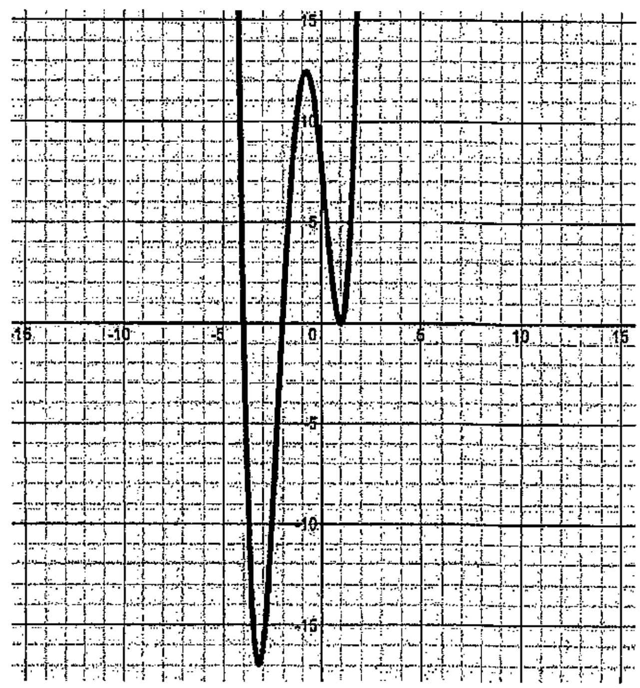

4. Given the graph below state the following information:

Zeroes: ____________ Degree: ____________ # of turns: ____________

Relative Maximum: ____________ Relative Minimum: ____________

Absolute Maximum: ____________ Absolute Minimum: ____________

End behavior: ________________ ________________

Decreasing Interval(s): ____________ Increasing Interval(s): ____________

Domain: ______________ Range: ______________

Real zeroes: ____________ Imaginary zeroes: ____________

Write the equation in factored form: ______________________________________

Write the equation in standard form: ______________________________________