

2/28 Wed	Review Factoring & Rational Numbers Pages 1-4	
3/1 Thurs	Unit 2 Polynomial Test	
3/2 Fri	Multiply & Divide of Rational Expressions Pages 5-7	Early Release
3/5 Mon	Multiply & Divide of Rational Expressions Pages 8-9	
3/6 Tues	Add & Subtract of Rational Expressions Pages 10-11	
3/7 Wed	Add & Subtract of Rational Expressions Pages 12-13	
3/8 Thurs	Simplifying Complex Fractions Pages 14-17	
3/9 Fri	Quiz (Simplify, Multiply, Divide, Add, and Subtract Rational Expressions)	
3/12 Mon	Solving Rational Equations Pages 18-20	
3/13 Tues	Application of Rational Equations Pages 21-25	
3/14 Wed	Quiz (Solve Rational Equations, Simplify Complex Fractions)	Pi Day!
3/15 Thurs	Graphing Rational Equations Pages 26-30	
3/16 Fri	Graphing Rational Equations Pages 31-35	
3/19 Mon	Review Pages 36-40	
3/20 Tues	Test	

Assignment

Date _____

Period _____

Factor each completely.

1) $40xy + 56x^2 - 5y - 7x$

2) $14uv + 35u + 12v + 30$

3) $40xy + 35x - 56y - 49$

4) $30xy - 40x^2 - 15y + 20x$

5) $9ab - 6a - 6b + 4$

6) $6xy + 30x^2 + 7y + 35x$

7) $x^2 - x - 90$

8) $2p^3 - 12p^2$

9) $b^4 - 3b^3 - 40b^2$

10) $x^3 + 17x^2 + 70x$

11) $3x^2 - 3x$

12) $4a^4 + 4a^3$

13) $21x^3 - 36x^2 + 15x$

14) $2b^2 - 27b + 81$

15) $10v^2 + 85v + 105$

16) $28n^2 + 52n - 96$

$$17) 3m^4 + 24m^2 + 36$$

$$18) 2x^4 - 32$$

$$2(x^4 - 16)$$

$$2(x^2 + 4)(x^2 - 4) =$$

$$2(x^2 + 4)(x + 2)(x - 2)$$

$$19) 648x^4 - 1029x^3$$

$$3x(216x^3 - 343)$$

$$3x(6x - 7)(36x^2 + 42x + 49)$$

$$20) -375a - 3a^4$$

$$21) 500x^4 + 256x$$

$$22) 216x^2 - 343x^5$$

$$23) 256x^4 - 500x$$

Solve each equation by factoring.

$$24) b^2 - b = 42$$

$$b^2 - b - 42 = 0$$

$$(\quad)(\quad) = 0$$

$$b = \underline{\quad} \quad b = \underline{\quad}$$

$$26) x^2 + 13x = -42$$

$$25) n^2 - 20 = -n$$

$$27) x^2 = -7x$$

$$28) 70x^2 + 64 = 152x$$

$$29) 5x^2 - 26x = -24$$

$$30) 3m^2 = -7m$$

$$31) 10k^2 = -24 + 46k$$

$$32) 2v^2 = 17v - 8$$

$$33) 16x^2 - 10 = -6x$$

4.3 What Does it Mean to Be Rational?

A Solidify Understanding Task

Part I: Comparing rational numbers and rational fractions.

1. In your own words, define *rational number*.

Circle the numbers below that are rational and refine your definition, if needed.

$$3 \quad -5 \quad \frac{2}{3} \quad \frac{20}{3} \quad 14 \quad 2.7 \quad \sqrt{5} \quad 2^3 \quad 3^{-3} \quad \log_2 8 \quad \frac{7}{0}$$

2. The definition of a *rational function* is as follows:

A function $f(x)$ is called a rational function if and only if it can be written in the form

$$f(x) = \frac{P(x)}{Q(x)} \text{ where } P \text{ and } Q \text{ are polynomials in } x \text{ and } Q \text{ is not the zero polynomial.}$$

Interpret this meaning in your own words and then write three examples of rational functions.

3. How are rational numbers and rational functions similar? Different?

Part II: Arithmetic of Rational Expressions: making connections between rational numbers and rational expressions. Solve problems in the first column and then use the same process to simplify the rational expressions in the second column.

Arithmetic of rational numbers	Arithmetic of rational expressions
4a. $\frac{2}{3} + \frac{4}{7}$	4b. $\frac{3}{(x+1)} + \frac{4}{(x-1)}$
5a. $\frac{3}{8} + \frac{5}{6}$	5b. $\frac{2x}{(x+3)} + \frac{4x}{(x-1)(x+3)}$

6a. $\frac{7}{8} - \frac{1}{6}$	6b. $\frac{2x}{(x+3)} - \frac{4}{(x-1)}$
7a. $\frac{3}{8} \times \frac{5}{6}$	7b. $\frac{(x+1)(x-2)}{(x+2)} \times \frac{(x+5)}{(x-2)(x+2)}$
8a. $\frac{3}{8} \div \frac{5}{6}$	8b. $\frac{(x+1)(x-2)}{(x+2)} \div \frac{(x+5)}{(x-2)(x+2)}$

9. To summarize, explain how you would perform the following arithmetic operations on rational expressions:

Adding:

Subtracting:

Multiplying:

Dividing:



Simplifying Rational Expressions

numerator _____
denominator _____

2

Descending Order

Descending Order

3

Factored Form

Factored Form

4

Reduce Common Terms

Reduce Common Terms

5

Simplified Form

Simplified Form

$x^2 + x^3 + 5 + x^4 + x$
 Changes to
 $x^4 + x^3 + x^2 + x + 5$

1. Factor out -1 if the first term is negative.
2. Factor out GCF
3. Factor using difference of squares, diamond, or box

- Reduce GCF's.
- Reduce pairs of parentheses
- If the entire numerator or denominator reduces write 1 in its place

1	2	3 and 4	5
$\frac{13x + 2x^2 + 20}{30 + 2x^2 + 17x}$	_____	_____	_____
$\frac{10x^2 + 20x + 10}{5x^2 - 5}$	_____	_____	_____
$\frac{15 + x - 6x^2}{3x^2 + 4x - 15}$	_____	_____	_____

CC3
Simplifying Rationals

Simplify.

1. $\frac{14x^5y^7}{28y^8}$

2. $\frac{12x^3y^7}{3x^4y^6}$

3. $\frac{4x^2(x+2)}{2x+4}$

4. $\frac{8x^5(3x-21)}{16x^7(x^2-49)}$

5. $\frac{27xy^3(x^2-1)}{3x^2y-3xy}$

6. $\frac{6-x}{x-6}$

7. $\frac{3x^2-3}{6x^2+12x+6}$

8. $\frac{-15-x+6x^2}{3x^2+4x-15}$

9. $\frac{54-6x-4x^2}{4x^2-81}$

CC3

Multiplying Rational Expressions

Simplify the following expressions.

1. $\frac{5a^4b}{12c} \cdot \frac{24bc^2}{15a^3b^2}$

2. $\frac{3}{4x+8} \cdot \frac{x^2+2x}{9}$

3. $\frac{x^2-y^2}{5x^3y^2} \div \frac{4x+4y}{15x^2y^5}$

4. $\frac{3p-21}{p^2-49} \cdot \frac{p^2+7p}{3p}$

5. $\frac{x^2-11x+24}{x^2-18x+80} \cdot \frac{x^2-15x+50}{x^2-9x+20}$

6. $\frac{3x^2+10x-8}{3x^2-17x+10} \div \frac{x^2+3x-4}{-2x^2+9x+5}$

7. $\frac{r^2+2r-8}{r^2+4r+3} \div \frac{r-2}{3r+3}$

8. $\frac{x^2+4x-32}{x^2-12x+35} \div \frac{x^2-4x-21}{-4x^2+16x} \cdot \frac{x^2-10x}{x^2+11x+24}$

9. $\frac{\frac{2x+6}{x^2+x-2}}{x+3}$
 $\frac{\quad}{x^2+3x+2}$

10. $\frac{\frac{t^3-t^2-2t}{t^2-3t+2}}{t^2+3t+2}$
 $\frac{\quad}{t^2+2t}$

CC3

Rational Expressions

A. Simplify

1. $\frac{15x^4}{5x^4}$

2. $\frac{7w^3}{28w^2}$

3. $\frac{16p^3}{24p^7}$

4. $\frac{9x-27}{9}$

5. $\frac{12x-15}{21}$

6. $\frac{4x-12}{4x+12}$

7. $\frac{x^2-16}{x^2-8x+16}$

8. $\frac{x^2-9x+8}{x^2+3x-4}$

9. $\frac{(x-y)}{x^2-y^2}$

B. Multiply and Simplify

10. $\frac{x^4}{3x+6} \cdot \frac{5x+10}{5x^7}$

11. $\frac{x^2-16}{2x+6} \cdot \frac{x+3}{x-4}$

12. $\frac{x^2+10x+25}{x^2-9} \cdot \frac{x^2-3x}{x+5}$

13. $\frac{x^2-y^2}{4x+4y} \cdot \frac{x+y}{x-y}$

14. $\frac{x^2-10x+9}{x^2-1} \cdot \frac{x+4}{x^2-5x-36}$

15. $\frac{(x-3)(x^2+3x+y^2)}{x^2-9} \cdot \frac{x^2-6x+9}{x^2+3x+9}$

C. Divide and Simplify

16. $\frac{12x^8}{3x^4} \div \frac{16x^3}{6x}$

17. $\frac{3x+15}{x} \div \frac{x+5}{x}$

18. $\frac{x^2-9}{x} \div \frac{x+3}{x+2}$

19. $\frac{x^2-16}{x^2-10x+25} \div \frac{3x-12}{x^2-3x-10}$

20. $\frac{x^2-3x-10}{2x^2-11x+5} \div \frac{x^2-5x+6}{2x^2-7x+3}$



Simplify, Multiply and Divide Rational Expressions

Name: _____

Date: _____

1. Simplify: $\frac{x^2 - 5x - 14}{x^2 - 49}$

- A. $\frac{x+2}{x+7}$ B. $\frac{2}{7}$ C. $\frac{x+2}{7-x}$ D. $\frac{x-2}{x-7}$

2. Simplify: $\frac{\left(\frac{7x^2y}{21x^2 - 6x}\right)}{\left(\frac{14x}{49x^2 - 4}\right)}$

- A. $\frac{6}{7xy + 2y}$ B. $\frac{7xy + 2xy}{6}$
 C. $\frac{6}{7xy + 2x}$ D. $\frac{7xy + 2y}{6}$

3. Simplify: $\frac{4a - 28}{6} \div \frac{a^2 - 49}{3a - 21}$

- A. $\frac{2(a-7)}{(a+7)}$ B. $\frac{2a-14}{7}$
 C. $\frac{a^2 + 4a - 77}{6(3a-21)}$ D. $\frac{a-8}{6(a-7)}$

4. Simplify: $\frac{3x^2 - 6x}{4 - x^2} \cdot \frac{3x^2 + 5x - 2}{27x^2 - 3}$

- A. $\frac{-x}{3x+1}$ B. $\frac{-x(x-2)}{(3x-1)(x+2)}$
 C. $\frac{x(x-2)}{(3x-1)(x+2)}$ D. $\frac{-x(x+2)}{(3x-1)(x+2)}$

5. Multiply: $\frac{x^2 + 3x - 10}{x^2 - 4} \cdot \frac{x+2}{x^2 - 9}$

- A. $\frac{5}{x-9}$ B. $\frac{x+5}{x-9}$
 C. $\frac{x+5}{x^2-9}$ D. $\frac{x-5}{x^2+9}$

6. Simplify: $\frac{\frac{2x+6}{x+1}}{\frac{x+3}{x^2-1}}$

- A. $2(x+1)$ B. $2(x-1)$
 C. $\frac{2(x+3)^2}{(x+1)^2(x-1)}$ D. $x+1$

7. Simplify: $\frac{c^2 - 3c - 10}{c^2 - 4} \div \frac{c^2 - 25}{c^2 + 2c - 15}$

- A. $\frac{c+2}{2(c-3)}$ B. $\frac{c-3}{c^2}$
 C. $\frac{c-3}{c-2}$ D. $\frac{c^2(c-3)}{c-2}$

8. Simplify: $\frac{2x^2 + 9x + 9}{x^2 - 6x - 27}$

- A. $\frac{2}{x-3}$ B. $-\frac{1}{3}$
 C. $\frac{2x+3}{x-9}$ D. does not simplify

9. Simplify: $\frac{x^2 - x - 6}{x^2 - 5x + 6}$

- A. -3 B. $\frac{x+2}{x-2}$ C. $\frac{x+3}{x+2}$ D. $\frac{x-2}{x+2}$

10. Simplify: $\frac{x^2 - x - 2}{x^2 + 6x + 5} \div \frac{2x^2 - 3x - 2}{4x^2 - 1}$

- A. $\frac{(x+1)(2x-1)}{(x+3)(x+2)}$ B. $\frac{2x-1}{x+5}$
 C. $\frac{x+5}{2x-1}$ D. $\frac{1}{(x+2)(x+3)}$

Honors Common Core III
Adding/Subtracting Rational Expressions

1. $\frac{2x+5}{5x+10} + \frac{x+1}{5x+10}$

2. $\frac{7a-10}{3a-12} - \frac{a+8}{3a-12}$

3. $\frac{6n+5}{2n+6} - \frac{2n-7}{2n+6}$

4. $\frac{2x^2-55}{x^2+7x} + \frac{6-x^2}{x^2+7x}$

5. $\frac{7a-15}{a^2-36} - \frac{2a+15}{a^2-36}$

6. $\frac{5c^2-8c}{c^2-9} + \frac{4c+9c^2}{c^2-9}$

7. $\frac{y^2-8y}{y^2+10y+16} + \frac{3y-14}{y^2+10y+16}$

8. $\frac{3n^2+11n}{n^2+16n+64} - \frac{n^2+40}{n^2+16n+64}$

9. $\frac{4a^2-16}{a^2-8a+16} - \frac{a^2+8a}{a^2-8a+16}$

10. $\frac{3x+7}{2x^2+10x+12} + \frac{x+5}{2x^2+10x+12}$

CC3

Add/Subtract Rational Expressions

1. $\frac{8}{3x^3y} + \frac{4}{9xy^3}$

2. $\frac{3y+1}{4y+4} - \frac{2y+7}{2y+2}$

3. $\frac{x+2}{5x-10} - \frac{3x+5}{2x-4}$

4. $\frac{7}{5y+25} - \frac{4}{3y+15}$

5. $\frac{x}{3x+9} + \frac{8}{x^2+3x}$

6. $\frac{5x}{2y+4} - \frac{6}{y^2+2y}$

7. $\frac{4}{x^2-9} + \frac{7}{x+3}$

8. $\frac{5}{x^2-36} + \frac{9}{x^2+5x-6}$

9. $\frac{2x}{x^2-x-2} - \frac{5x}{x^2-3x+2}$

10. $\frac{y}{y^2-y-6} - \frac{y+2}{y^2+5y+6}$

11. $3x + \frac{x^2+5x}{x^2-2}$

12. $\frac{5y}{y^2-7y} - \frac{4}{2y-14} + \frac{9}{y}$

CC3

Rational Expressions

Simplify the following expressions

1. $\frac{4}{3a} + \frac{8}{3a}$

2. $\frac{3}{4a^2b} - \frac{7}{4a^2b}$

3. $\frac{a-5b}{a+b} + \frac{a+7b}{a+b}$

4. $\frac{4y+2}{y-2} - \frac{y-3}{y-2}$

5. $\frac{3a-2}{a^2-25} - \frac{4a-7}{a^2-25}$

6. $\frac{2a-5}{a^2-9} - \frac{3a-8}{a^2-9}$

Simplify the following expressions by factoring out a negative 1.

7. $\frac{a^2}{a-b} + \frac{b^2}{b-a}$

8. $\frac{x-7}{x^2-16} - \frac{x-1}{16-x^2}$

9. $\frac{t^2+3}{t^4-16} + \frac{7}{16-t^4}$

Find common denominators for the following expressions.

10. $\frac{a+2}{a-4} + \frac{a-2}{a+3}$

11. $\frac{a+3}{a-5} + \frac{a-2}{a+4}$

12. $\frac{8}{2x^2-7x+5} + \frac{3x+2}{2x^2-x-10}$

13. $\frac{-x}{x^2+9x+20} - \frac{4}{x^2+7x+12}$

14. $\frac{2x+1}{x-y} + \frac{5x^2-5xy}{x^2-2xy+y^2}$
 $\frac{5x(x-y)}{(x-y)(x-y)}$

15. $\frac{x-3}{x^2-16} - \frac{3x-2}{x^2+2x-24}$

Add and Subtract Rational Expressions

Name: _____

Date: _____

1. Simplify: $\frac{2x+5}{3} - \frac{5}{x}$

A. $\frac{2x^2+5x-15}{3x}$

B. $\frac{2}{3}$

C. $\frac{7x-15}{3x}$

D. $\frac{2x}{3-x}$

2. Simplify: $\frac{x-3}{x+3} + \frac{6}{x-3}$

A. $\frac{x^2+27}{x^2-9}$

B. $\frac{1}{x^2-9}$

C. $\frac{2x-6}{x^2-9}$

D. $\frac{2x^2-18}{x^2+9}$

3. Simplify: $\frac{5x}{x-1} - \frac{2x}{x-2}$

A. $\frac{3x^2-8x}{(x-1)(x-2)}$

B. $\frac{x^2-3x+2}{(x-1)(x-2)}$

C. $\frac{3x^2+8x}{(x-1)(x-2)}$

D. $\frac{3x^2+12x}{(x-1)(x-2)}$

4. Add: $\frac{5}{2x-8} + \frac{3x}{x^2-16}$

A. $\frac{11x}{2(x+4)(x-4)}$

B. $\frac{11x+4}{2(x+4)(x-4)}$

C. $\frac{11x+20}{2(x+4)(x-4)}$

D. $11x+20$

5. Simplify: $\frac{6n-3}{n} - \frac{2n+4}{n}$

A. $\frac{4n+1}{n}$

B. 3

C. $4n+1$

D. $\frac{4n-7}{n}$

6. Simplify: $\frac{x-3}{x+2} + \frac{5}{x-2}$

A. $\frac{x^2+16}{x^2-4}$

B. $\frac{1}{x-2}$

C. $\frac{x+2}{2x}$

D. $\frac{x+2}{x^2-4}$

7. Subtract and simplify: $\frac{2x+8}{x^2+6x+8} - \frac{x+16}{x^2+8x+12}$

A. $\frac{x-8}{-2x-4}$

B. $\frac{x-4}{(x+2)(x+6)}$

C. $\frac{x-8}{(x+4)(x+6)}$

D. $\frac{-x-14}{(x+2)(x+6)}$

(13)

CC3

Complex Rational Expressions

$$1. \frac{\frac{2}{x}}{\frac{9}{y}}$$

$$2. \frac{-5}{\frac{4}{x} + y}$$

$$3. \frac{\frac{7}{x} + \frac{2}{y}}{\frac{-6}{x} + \frac{3}{y}}$$

$$4. \frac{\frac{4}{xy} - \frac{1}{y}}{\frac{2}{x} + \frac{3}{y}}$$

$$5. \frac{\frac{7}{x+1} + 3}{2 + \frac{5}{x+1}}$$

$$6. \frac{\frac{6}{y+7}}{\frac{8y}{y^2 - 49}}$$

$$7. \frac{\frac{5}{4y-4}}{\frac{3y}{y^2-1}}$$

Name _____

Complex Fractions

Simplify:

$$1. \frac{\frac{7}{x^2-4} + \frac{2}{x-2}}{\frac{6}{x-2} + \frac{5}{x+2}} =$$

$$2. \frac{\frac{4}{x^2-6x-16} - \frac{3}{5x-40}}{\frac{5}{2x-16} + \frac{3}{x+2}} =$$

$$3. \frac{1-25a^{-2}}{1-3a^{-1}-10a^{-2}} =$$

$$4. \frac{\frac{2}{x} - \frac{10}{x^2+7x}}{\frac{5}{x+7} + \frac{2}{3x}} =$$

$$5. \frac{\frac{3}{a} - \frac{9}{a^2+3a}}{\frac{4}{a+3} - \frac{1}{a}} =$$

$$6. \frac{1-2x^{-1}+x^{-2}}{1+2x^{-1}-3x^{-2}} =$$

Honors Math 3
Complex Rational Expressions

Simplify.

1. $\frac{\frac{4n}{9n^3}}{\frac{8n^4}{15n^2}}$

3. $\frac{\frac{7}{x} - 6}{\frac{2}{x} + 3}$

5. $\frac{a^2 + \frac{2a}{3}}{\frac{a^2}{6} + 14a}$

7. $\frac{\frac{2x}{5} + \frac{3y}{10}}{\frac{3x}{2} - \frac{7y}{10}}$

9. $\frac{1 + \frac{1}{2a}}{\frac{1}{4a} - a}$

2. $\frac{\frac{1}{a} - a}{\frac{3}{a}}$

4. $\frac{\frac{2}{7x} - \frac{3}{14}}{\frac{3}{14x} - \frac{1}{7}}$

6. $\frac{\frac{2}{c} + \frac{1}{2c}}{c + \frac{c}{2}}$

8. $\frac{\frac{c}{3d} - \frac{d}{3c}}{\frac{1}{3c} + \frac{1}{3d}}$

10. $\frac{y + 4 + \frac{4}{y}}{y + 1 - \frac{2}{y}}$

11. $\frac{9u^4 - \frac{1}{w^2}}{3u^2 + \frac{1}{w}}$

13. $\frac{\frac{5}{x+y} + 3}{\frac{3}{x+y} + 7}$

15. $\frac{\frac{n}{n-2} - \frac{2}{n+2} - \frac{8}{n^2-4}}{\frac{1}{n-2}}$

12. $\frac{\frac{w}{w-z} - 1}{\frac{w}{w-z}}$

14. $\frac{u - 2 - \frac{4}{u-2}}{u - 1 - \frac{2}{u-2}}$

16. $\frac{\frac{3a}{a^2-9}}{\frac{1}{3-a} - \frac{1}{3+a}}$

17. $\frac{9 + \frac{6}{c} + \frac{1}{c^2}}{\frac{3}{c} + \frac{1}{c^2}}$

19. $2 - \frac{1}{2 + \frac{1}{\frac{1}{n}}}$

21. $\frac{1}{1 - \frac{1}{2 - \frac{1}{3 - \frac{1}{4}}}}$

18. $\frac{\frac{1}{p+1} - 1}{\frac{1}{p^2-1} + 1}$

20. $2 + \frac{1}{2 + \frac{1}{2+x}}$

22. $\frac{\frac{6}{a-2} + \frac{5}{a+2}}{\frac{7}{a^2-4} + \frac{2}{a-2}}$

23. $\frac{\frac{y^2+8y+15}{y^2+y-6}}{\frac{y^2+2y-15}{y^2-2y-3}}$

25. $\frac{x - 2 + \frac{9x+11}{x+3}}{x + 5 + \frac{x-15}{x+4}}$

24. $\frac{\frac{p+4}{p-6} + \frac{p+1}{p+2}}{\frac{2p^2+3}{p^2-4p-12}}$

26. $\frac{u - 4}{u - 1 - \frac{15}{u+1}}$

Math 3 Unit 7 Quiz Review

Name: _____

Date: _____

1. Simplify: $\frac{12x^2 + 4x}{9 - 4x^2} \cdot \frac{6x^2 + 7x - 3}{18x^2 - 2}$

A. $\frac{-2x(3x+1)}{(2x+3)(3x-1)}$ B. $\frac{2x}{2x-3}$

C. $\frac{-2x}{2x-3}$ D. $\frac{-2x}{2x+3}$

2. Simplify: $\frac{5x}{x-1} - \frac{2x}{x-2}$

A. $\frac{3x^2 - 8x}{(x-1)(x-2)}$ B. $\frac{x^2 - 3x + 2}{(x-1)(x-2)}$

C. $\frac{7x^2 + 12x}{(x-1)(x-2)}$ D. $\frac{3x^2 + 12x}{(x-1)(x-2)}$

3. Add: $\frac{5}{2x-8} + \frac{3x}{x^2-16}$

A. $\frac{11x}{2(x-4)^2}$ B. $\frac{11x}{2(x+4)(x-4)}$

C. $\frac{11x+4}{2(x+4)(x-4)}$ D. $\frac{11x+20}{2(x+4)(x-4)}$

4. Express $\frac{\frac{y}{y+3}}{1 - \frac{y}{y+3}}$ in simplest form.

5. Multiply: $\frac{x^2 + 3x - 10}{x^2 - 4} \cdot \frac{x+2}{x^2 - 9}$

A. $\frac{5}{x-9}$ B. $\frac{x+5}{x-9}$

C. $\frac{x+5}{x^2-9}$ D. $\frac{x-5}{x^2+9}$

6. Simplify: $\frac{x^2 + 4x - 21}{x^2 - 6x + 9} \div \frac{x+7}{x-3}$

A. 1 B. -1

C. $\frac{(x-3)^2}{(x+7)^2}$ D. $\frac{x+7}{x-3}$

7. Subtract and simplify: $\frac{2x+8}{x^2+6x+8} - \frac{x+16}{x^2+8x+12}$

A. $\frac{x-8}{-2x-4}$ B. $\frac{x-4}{(x+2)(x+6)}$

C. $\frac{x-8}{(x+4)(x+6)}$ D. $\frac{-x-14}{(x+2)(x+6)}$

8. Simplify: $\frac{4a-28}{6} \div \frac{a^2-49}{3a-21}$

A. $\frac{2(a-7)}{(a+7)}$ B. $\frac{2a-14}{7}$

C. $\frac{a^2+4a-77}{6(3a-21)}$ D. $\frac{a-8}{6(a-7)}$

17

CC3

Solving Rational Equations

1. $\frac{1}{3} + \frac{4}{5} = \frac{x}{9}$

2. $\frac{7}{8} + \frac{2}{5} = \frac{x}{20}$

3. $\frac{1}{t} + \frac{1}{3} = \frac{8}{3t}$

4. $\frac{1}{x} - \frac{5}{6x} = \frac{2}{3}$

5. $\frac{3}{2} - \frac{3}{x} = \frac{9}{2x}$

6. $\frac{1}{3z} + \frac{1}{8} = \frac{4}{3z}$

7. $\frac{1}{4x} - \frac{3}{4} = \frac{7}{x}$

8. $\frac{2}{y} + \frac{1}{2} = \frac{2}{2y}$

9. $\frac{4}{3n} - \frac{3}{n} = \frac{10}{3}$

10. $\frac{1}{2} - \frac{2}{7} = \frac{3}{2x}$

CC3
Rational Equations

Solve each rational equation.

1. $\frac{3x}{4} = \frac{5x+1}{3}$

2. $5 - \frac{4}{x+1} = 6$

3. $\frac{x}{x+3} - \frac{x}{x-3} = \frac{x^2+9}{x^2-9}$

4. $\frac{7}{x^2-5x} + \frac{2}{x} = \frac{3}{2x-10}$

5. $\frac{x+3}{x^2+3x-4} = \frac{x+2}{x^2-16}$

6. $\frac{4}{x-3} = \frac{2}{x+1} + \frac{16}{x^2-2x-3}$

Name _____

Rational Equations

Solve each equation. Write the final answer in a solution set. Show your work.

1. $\frac{9}{x-3} = \frac{x-4}{x-3} + \frac{1}{4}$

2. $\frac{3}{b^2+5b+6} + \frac{b-1}{b+2} = \frac{7}{b+3}$

3. $\frac{1}{n-2} = \frac{2n+1}{n^2+2n-8} + \frac{2}{n+4}$

4. $\frac{4x^2}{x^2-9} - \frac{2x}{x+3} = \frac{3}{x-3}$

5. $\frac{5z+2}{z^2-4} = \frac{-5z}{2-z} + \frac{2}{z+2}$

6. $\frac{a-4}{a+3} = \frac{3a+2}{a+3} + \frac{a}{4}$

Your family LOVES coffee! You decided you wanted to get a Keurig machine but your parents don't want to spend the money. Instead they splurge on Starbucks a couple of times a week. Because you like getting your way, you decided to use MATH in order to convince your parents you are right (as always).



Find the 3 beverages your "family" typically get at Starbucks and compute the average.

1) _____

2) _____

3) _____

AVERAGE COST: _____

Now, go online and find a good deal on 3 Keurig machines that you wish to purchase. Make sure you purchase from a reputable store. (Circle the Keurig you decide to get)

Store	Price
_____	_____
_____	_____

The cost of cream and sugar can be negated by the cost of gas, but we need to find the cost of a single k-cup. Shop online and find 3 places that sell tasty k-cups and write down the cost per one k-cup.

Store	Price
_____	_____
_____	_____

Cost of Machine: _____ Cost of 1 k-cup: _____

Find the average price you would pay per cup of coffee

Cups Avg Price(per cup)

1	
2	
3	
4	
5	
6	
10	
25	



What equation could we write that could give us the cost per "n" cups of coffee?

How many cups of Keurig Coffee would we have to make in order for it to be cheaper than going to Starbucks?

What is the cheapest you'll ever pay for a cup of coffee? Explain...

Rate of Work Examples-Notes

1. One computer system can prepare the weekly sales summary of a company in 10 hours. A faster system can do the job in 6 hours. How many hours would it take both systems, working together, to prepare the summary?
2. Working together, Tina, Tony, and Andrea can paint a house in 35 hours. Andrea can do the job alone in 70 hours. Tina can do the job alone in twice the time it would take Tony working alone. Find the time it would take Tony working alone.
3. Machine A can do a job in 12 hours and Machine B can do the job in 8 hours. If B starts 2 hours after A, find the total time needed for the two machines to do the complete job.

RATE OF WORK PROBLEMS

NAME _____

1. Mr. Adams can plant a wheat crop in 10 days and his daughter can do it in 15 days. How many days will it take if they work together?
2. Paul can put carpet on a floor in 10 hours. If Irene helps him, the job is done in 6 hours. How long would it take Irene if she worked alone?
3. Machine A can do a job in 15 hours. Machines B and C can do the same job in 12 hours and 20 hours, respectively. How many hours will the job take if the three machines operate at the same time?
4. Kim can complete a job in 6 weeks and the same job would take Kevin 10 weeks. How long would it take Derek working alone if, working together, all three can complete the job in 2 weeks?
5. It takes Lois 3 times as long as Richie to mow a lawn. How long would it take each of them alone, if together they can do it in 5 hours?
6. Working together, Andy and Sal can build a fence in 7 hours. Alone, it takes Andy twice the time it takes Sal. How long does it take each working alone?
7. Holly can harvest a strawberry patch in 5 hours, and Evelyn can do the job in 8 hours. Given that Evelyn starts 2 hours after Holly has begun working, find the total time needed to do the job.
8. Steve can mow a lawn in 75 minutes and Chuck can do it in 50 minutes. If Chuck watches Steve mow for 20 minutes and then helps to finish the job, find the total time for the job.
9. A large pipe can fill a tank in 5 hours, and a smaller one can fill the tank in 8 hours. A drain pipe can empty the tank in 10 hours. Find the total time to fill the tank when all three pipes are left open.

Rational Function Applications

Name: _____

Date : _____

1. A rare species of insect was discovered in the Amazon Rain Forest. To protect the species, environmentalists declare the insect endangered and transplant the insects into a protected area. The population of the insect t months after being transplanted is given by

$$P(t) = \frac{50(1 + 0.5t)}{(2 + 0.01t)}$$

- Graph the function and find a good window.
- How many insects were discovered? In other words, what was the population when $t = 0$?
- What will the population be after 5 years?
- Determine the horizontal asymptote. What is the largest population that the protected area can sustain?

Go to 2nd Window (this is the table setup menu)

Edit TBLStart = and set it = to 1000

Go to 2nd Y = (This brings up the table and it now starts at 1000)

Repeat these steps for 10000 and 100000

What is y approaching as x gets bigger and bigger? (This is the horizontal asymptote)

* Change your tblstart so it is back at 1

2. The model $N(t) = \frac{10(4 + 3t)}{1 + 0.02t}$ describes the number of deer (N) in a park t years after 40 deer are introduced into the region. Approximately how long (to the nearest year) will it take the deer population to reach 196?

What is the largest population of deer the park can sustain? (Find the horizontal asymptote)

3. The concentration C of a certain drug in a patient's bloodstream t hours after injection is given

by:
$$C(t) = \frac{t}{2t^2 + 1}$$

- Graph the function
- Determine the time at which the concentration is highest.
- Find the horizontal asymptote of $C(t)$. What happens to the concentration of the drug as t increases?

4. The function $C(x) = \frac{100x + 1,000}{x}$ models the average cost per bicycle, $C(x)$, for a manufacturing business that produces x bicycles. How many bicycles must be manufactured to bring the average cost per bicycle down to \$125?

5. The function $t(x) = \frac{40}{x} + \frac{40}{x + 30}$ models the total time t required to complete a round trip that is 40 miles in each direction, where

X = the average speed on the outgoing trip

$X + 30$ = the average speed on the return trip

What outgoing speed will result in the round trip taking 5 hours?

Original Equation $\rightarrow \frac{x-1}{2x^2-x-1} = \frac{(x-1)}{(2x+1)(x-1)}$ Factored Form

Y-intercept (0,1) X-intercept None (all numerator terms reduced)

Horizontal Asymptote $y=0$ Vertical Asymptote $x=-\frac{1}{2}$ Domain $x \neq -\frac{1}{2}, x \neq 1$ Hole $x=1$

Original Equation $\rightarrow \frac{x+2}{x^2+5x+6} = \frac{\quad}{\quad}$ Factored Form

Y-intercept X-intercept

Horizontal Asymptote Vertical Asymptote Domain Hole

Original Equation $\rightarrow \frac{x-3}{2x^2-5x-3} = \frac{\quad}{\quad}$ Factored Form

Y-intercept X-intercept

Horizontal Asymptote Vertical Asymptote Domain Hole

Original Equation $\rightarrow \frac{x^2 - 7x - 8}{x^2 - 3x + 2} = \frac{(x-8)(x+1)}{(x-2)(x-1)}$ Factored Form

Y-intercept (0,-4) X-intercept (8,0) and (-1,0)

Horizontal Asymptote $y = 1$ Vertical Asymptote $x = 2$ and $x = 1$ Domain $x \neq 1, x \neq 2$ Hole (No terms reduced)

Original Equation $\rightarrow \frac{x^2 - 6x + 8}{x^2 + 6x + 8} = \frac{\quad}{\quad}$ Factored Form

Y-intercept X-intercept

Horizontal Asymptote Vertical Asymptote Domain Hole

Y-intercept X-intercept

Original Equation $\rightarrow \frac{2x^2 + 3x - 5}{x + 2} = \frac{\quad}{\quad}$ Factored Form

Horizontal Asymptote Vertical Asymptote Domain Hole

Rational Functions

Name: _____
Date: _____

	Y-intercept	X-intercept	
Original Equation	$\frac{x^2 - 16}{x^2 - 7x + 12}$	=	Factored Form
Horizontal Asymptote	Vertical Asymptote	Domain	Hole

	Y-intercept	X-intercept	
Original Equation	$\frac{3x^2 - 2x - 1}{x + 4}$	=	Factored Form
Horizontal Asymptote	Vertical Asymptote	Domain	Hole

	Y-intercept	X-intercept	
Original Equation	$\frac{x - 3}{x^2 + 3x - 18}$	=	Factored Form
Horizontal Asymptote	Vertical Asymptote	Domain	Hole

Rational Functions

Name: _____

Date : _____

	Y-intercept		X-intercept	
Original Equation	$\frac{4x}{x^3 - 4x} = \underline{\hspace{2cm}}$			Factored Form

Horizontal Asymptote	Vertical Asymptote	Domain	Hole

	Y-intercept		X-intercept	
Original Equation	$\frac{x^2 + 6x + 9}{x + 3} = \underline{\hspace{2cm}}$			Factored Form

Horizontal Asymptote	Vertical Asymptote	Domain	Hole

	Y-intercept		X-intercept	
Original Equation	$\frac{4x^2 - 100}{2x^2 + x - 15} = \underline{\hspace{2cm}}$			Factored Form

Horizontal Asymptote	Vertical Asymptote	Domain	Hole

CC3

Graphing rational functions

Identify the critical features of each of the functions below. Check by graphing on your calculator.

1. $f(x) = \frac{3x^2 - 12x - 15}{3x + 6}$

2. $f(x) = \frac{x^3 + 3x^2}{x^4 - 4x^2}$

3. $f(x) = \frac{x^2 - 1}{x^3 - 2x^2 + x}$

4. $f(x) = \frac{x^4 - 1}{x^2}$

4.6 Features of Rational Functions

A Solidify Understanding Task

Part I: Identifying features of rational functions.

Using prior knowledge of other functions you have worked with, determine the features of each rational function and then sketch a graph. (features include intercepts, domain, asymptotes, end behavior*, and any other features you notice about each rational function).

$$1. f(x) = \frac{(x+1)}{(x-2)(x+2)}$$

x-intercept(s):

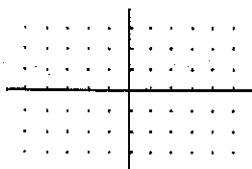
y-intercept:

Vertical asymptote(s):

Proper or Improper:

End behavior
asymptotes*:

Sketch the features on
the graph below:



Create a Sign Line, then
complete graph above:



$$2. f(x) = \frac{x+4}{(x+5)(x+1)}$$

x-intercept(s):

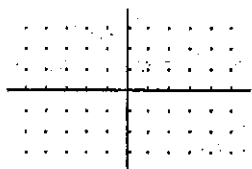
y-intercept:

Vertical asymptote(s):

Proper or Improper:

End behavior
asymptotes*:

Sketch the features on
the graph below:



Create a Sign Line, then
complete graph above:



$$3. f(x) = \frac{(x+3)(x+2)}{x+5}$$

x-intercept(s):

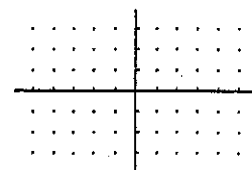
y-intercept:

Vertical asymptote(s):

Proper or Improper:

End behavior
asymptotes*:

Sketch the features on
the graph below:



Create a Sign Line, then
complete graph above:



Graphing Rational Expressions

Example 1: Simplify the following. State any restrictions on the variables.

a) $\frac{(x+1)(x-5)}{(x-5)(x^2-1)}$

b) $\frac{x^2+x-12}{x^2+7x+12}$

Vertical Asymptotes: Where the _____ of a function equals zero.

Point of Discontinuity: A _____ in the graph.

Example 2: Determine the equations of any vertical asymptotes and the values of x for any holes in the

graph of $f(x) = \frac{x^2-1}{x^2-6x+5}$.

Example 3: Determine the equations of any vertical asymptotes and the values of x for any holes in the

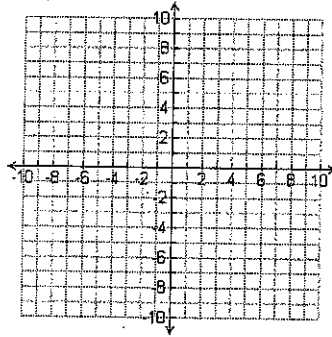
graph of $f(x) = \frac{x^2-4}{x^2+5x+6}$.

Horizontal Asymptotes: determined by comparing the degree of the numerator to the degree of the denominator. Let m = degree of numerator and n = degree of denominator.

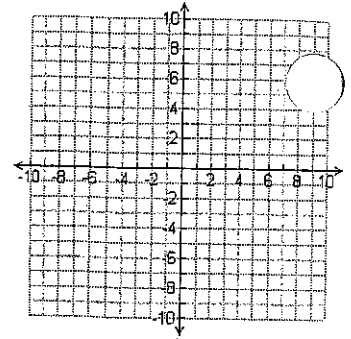
If...	Then the graph has...
$m < n$ $f(x) = \frac{x+4}{x^2+5x+4}$	A horizontal asymptote at $y = 0$ V.A.: _____ Hole(s): _____ H.A.: _____ Domain: _____
$m = n$ $f(x) = \frac{x^2+5x+4}{4x^2-9}$	A horizontal asymptote at the coefficient of m divided by the coefficient of n V.A.: _____ Hole(s): _____ H.A.: _____ Domain: _____
$m > n$ $f(x) = \frac{x^2+5x+4}{x+4}$	No horizontal asymptote V.A.: _____ Hole(s): _____ H.A.: _____ Domain: _____

Example 4: State the asymptotes and points of discontinuity of each equation, and then graph the function and state the domain.

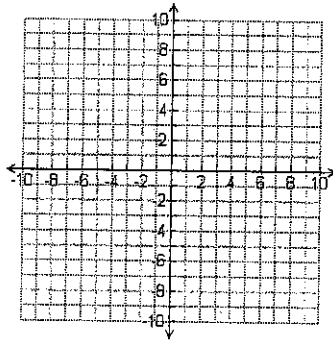
a) $f(x) = \frac{x^2 + x - 2}{x - 1}$



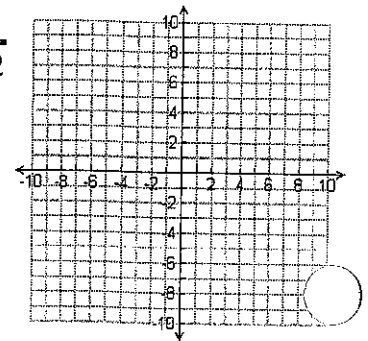
b) $f(x) = \frac{2x^2 + 3}{x + 2}$



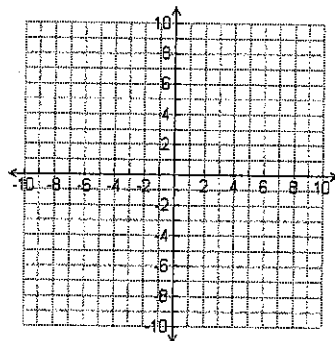
c) $f(x) = \frac{x - 1}{x^2 - 1}$



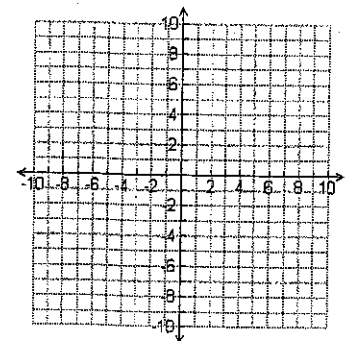
d) $f(x) = \frac{x - 3}{x^2 - 7x + 12}$



e) $f(x) = \frac{x^2 + 10x + 25}{x^2 + 9x + 20}$



f) $f(x) = \frac{x^2 + 12x + 36}{x^2 - 36}$



Graphing Rational Functions

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each.

1) $f(x) = \frac{1}{3x^2 + 3x - 18}$

2) $f(x) = \frac{x-2}{x-4}$

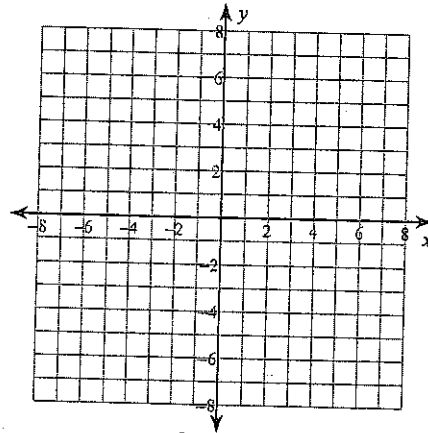
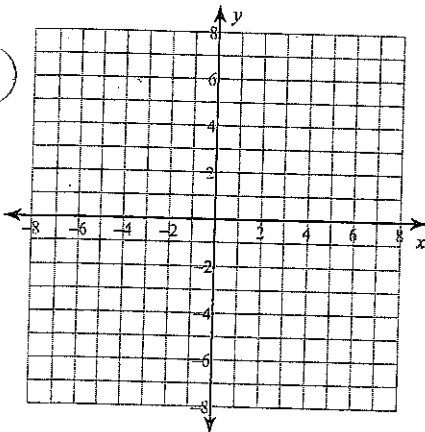
3) $f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$

4) $f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$

Identify the points of discontinuity, holes, vertical asymptotes, and horizontal asymptote of each. Then sketch the graph.

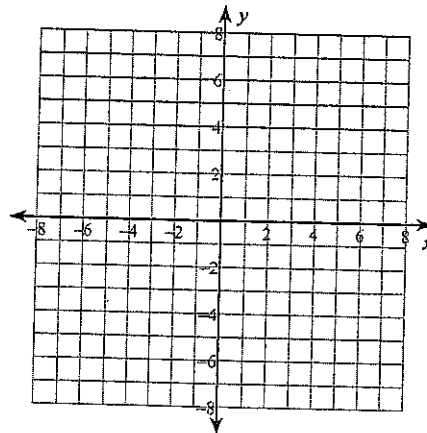
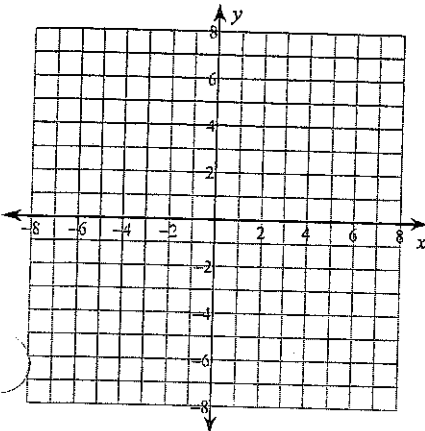
5) $f(x) = -\frac{4}{x^2 - 3x}$

6) $f(x) = \frac{x-4}{-4x-16}$

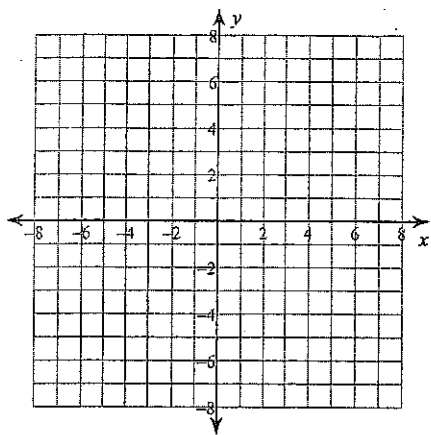


7) $f(x) = \frac{x+4}{-2x-6}$

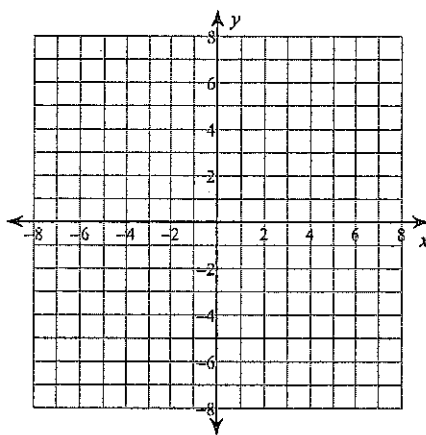
8) $f(x) = \frac{x^3 - 9x}{3x^2 - 6x - 9}$



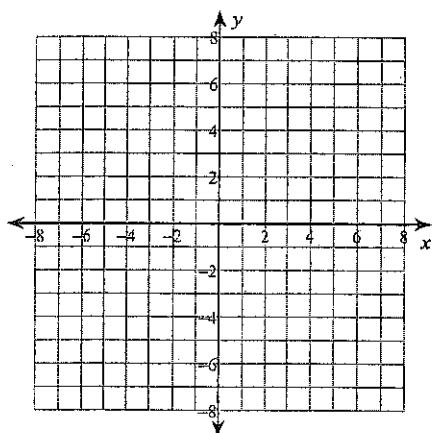
$$9) f(x) = \frac{3x^2 - 12x}{x^2 - 2x - 3}$$



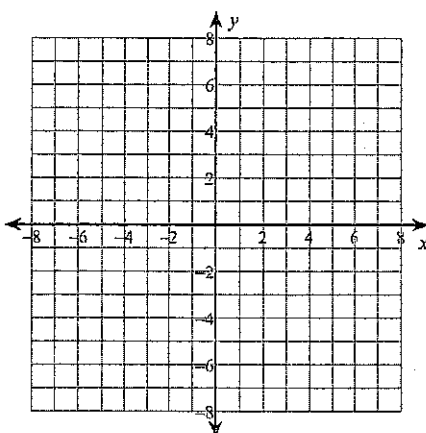
$$10) f(x) = \frac{x^3 - 16x}{-4x^2 + 4x + 24}$$



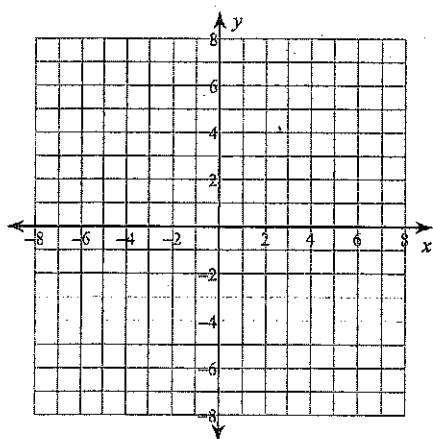
$$11) f(x) = \frac{x^2 + 2x}{-4x + 8}$$



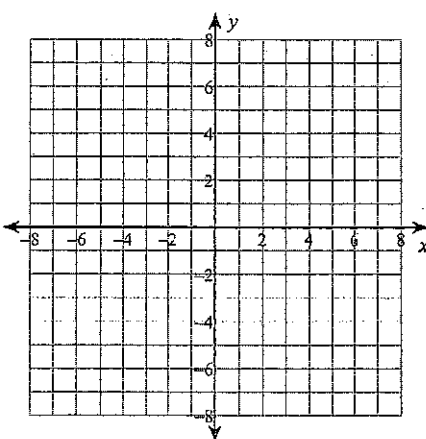
$$12) f(x) = \frac{x + 2}{2x + 6}$$



$$13) f(x) = \frac{2x^2 + 10x + 12}{x^2 + 3x + 2}$$



$$14) f(x) = \frac{3}{x - 2}$$



Warm Up: Rational Functions

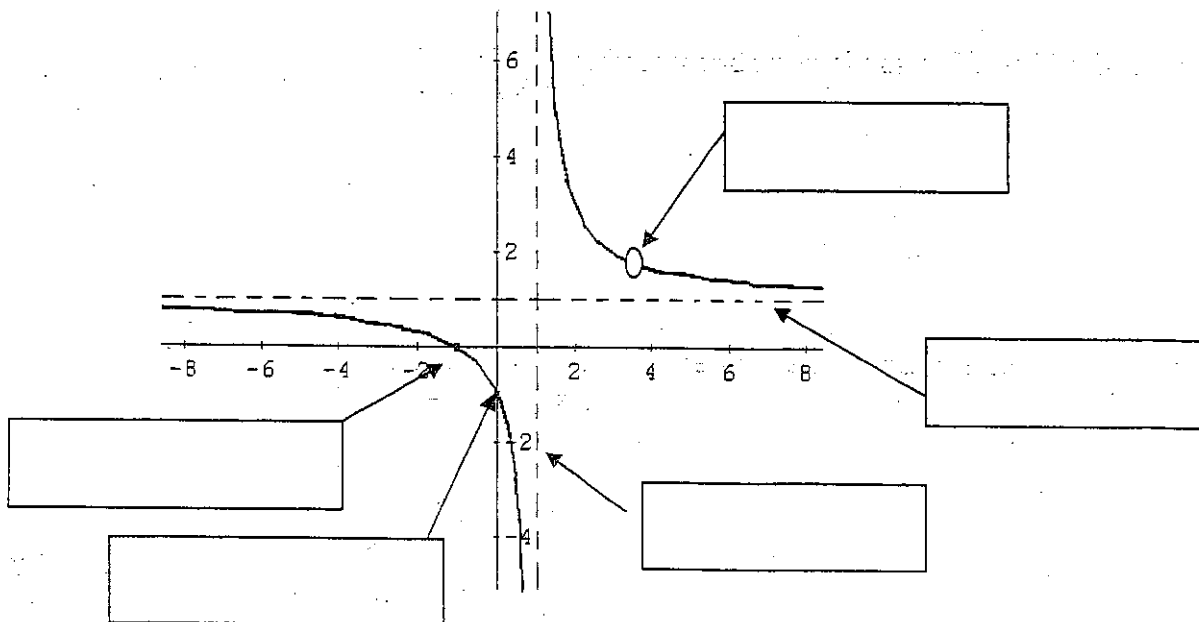
Name: _____

Date : _____

1. Give all transformations, domain, vertical and horizontal asymptotes for $y = \frac{3}{x+4} - 7$.
2. Identify all of the pieces of the function below

	Y-intercept		X-intercept		
Original Equation	$\frac{x^2 - 3x - 4}{x^2 - 5x + 4} =$			Factored Form	
Horizontal Asymptote	Vertical Asymptote	Domain	Hole		

3. Fill in the boxes below with the **term** (not a number) that identifies each part of the graph.



Assignment

Date _____ Period _____

Simplify each expression.

1) $\frac{35p+30}{5} \div \frac{21p+18}{6}$

2) $\frac{x^2+2x-24}{x^2+4x-5} \cdot \frac{1}{x+6}$

3) $\frac{6n^3-48n^2}{6n^2} \div \frac{n^2-16n+64}{5n}$

4) $\frac{x^2-13x+42}{x-7} \div \frac{6-x}{7x^2}$

5) $\frac{6x^2-18x}{x-7} \cdot \frac{1}{6x}$

6) $\frac{x+1}{x-4} \div \frac{6}{6x^3-24x^2}$

7) $\frac{8x^2-16x}{16x-8x^2} \cdot \frac{x+8}{5}$

8) $\frac{6p^3-24p^2}{7p} \cdot \frac{p+6}{p^2+2p-24}$

9) $\frac{7}{p-8} - \frac{p+7}{p-4}$

10) $\frac{4x-3}{7x+2} - \frac{3x-5}{5x^2}$

11) $\frac{3x}{5x+5} - \frac{4}{5x}$

12) $\frac{8}{3k} + \frac{5k-8}{k-6}$

13) $7 + \frac{a-6}{40a+24}$

14) $\frac{x-1}{7x-7} - \frac{5}{5x^2}$

15) $7n - \frac{3n-7}{n^2-9}$

16) $\frac{6}{n-7} + \frac{3n}{3n-5}$

Solve each equation. Remember to check for extraneous solutions.

$$17) \frac{x^2}{x^2 - 5x - 14} + \frac{1}{x^2 - 5x - 14} = \frac{x + 2}{x - 7}$$

$$18) 6r + 24 = \frac{5r + 40}{6r} + \frac{8r^2 - 35r + 12}{6r}$$

$$19) \frac{1}{x^2 + 4x - 21} + 1 = \frac{x - 1}{x - 3}$$

$$20) \frac{1}{7x - 1} - \frac{3x^2 - 15x - 18}{7x^3 - x^2} = \frac{1}{7x^3 - x^2}$$

$$21) \frac{n - 5}{n + 1} = \frac{n - 8}{n + 1} + \frac{1}{n^2 + n}$$

$$22) \frac{1}{a^2 + 3a} - \frac{a + 1}{a} = \frac{a - 2}{a^2 + 3a}$$

$$23) \frac{v - 2}{v} + \frac{v - 3}{v - 6} = \frac{1}{v}$$

$$24) \frac{1}{b + 1} = \frac{b^2 - b - 12}{b^2 + b} - 1$$

Simplify each expression.

$$25) \frac{\frac{x}{16} - \frac{4}{x}}{\frac{x^2}{16}}$$

$$26) \frac{\frac{3}{m + 4} - \frac{3}{12}}{m + 4}$$

$$27) \frac{\frac{5a}{2}}{\frac{1}{2} - \frac{5}{a}}$$

$$28) \frac{\frac{1}{3} + \frac{36}{x}}{x}$$

$$29) \frac{\frac{m^2}{6} - \frac{m}{4}}{m}$$

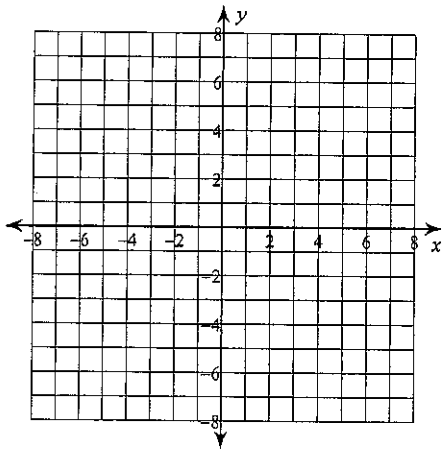
$$30) \frac{\frac{16}{u - 1}}{\frac{u - 1}{16} - \frac{u - 1}{4u}}$$

$$31) \frac{6x + 36}{\frac{1}{x} + \frac{36}{x+6}}$$

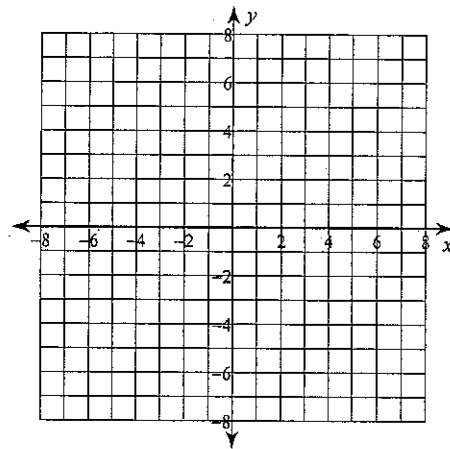
$$32) \frac{\frac{4}{x}}{\frac{1}{4} - \frac{1}{2}}$$

Identify the holes, vertical asymptotes, and horizontal asymptote of each. Then sketch the graph.

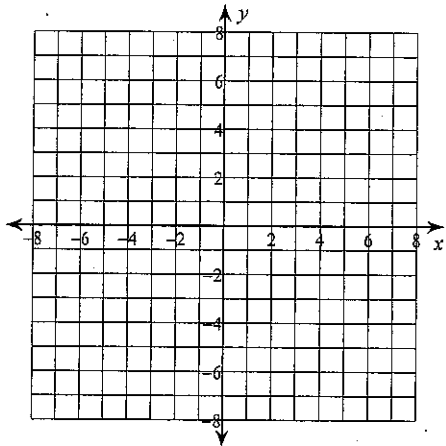
$$33) f(x) = \frac{x^2 + x - 12}{-2x - 4}$$



$$34) f(x) = \frac{1}{3x - 12}$$

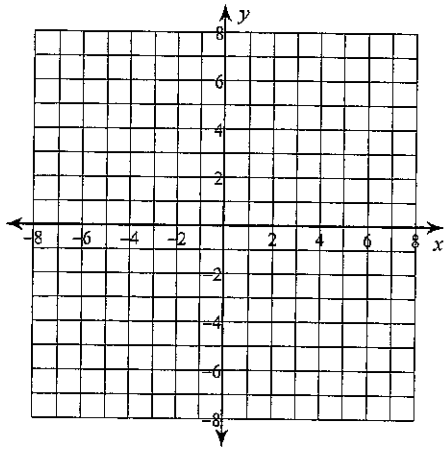


$$35) f(x) = \frac{1}{3x^2 - 3x - 18}$$

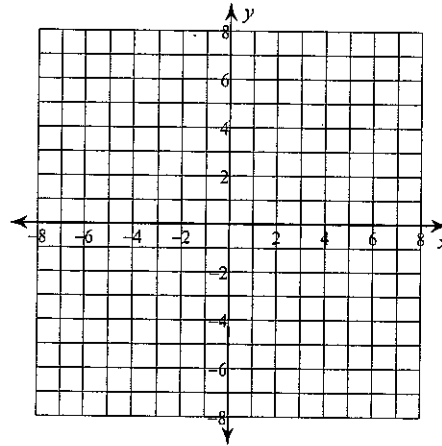


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$$36) f(x) = \frac{x^2 - x - 12}{-3x - 6}$$



$$37) f(x) = \frac{x + 4}{-4x + 8}$$



$$38) f(x) = \frac{x^3 + 4x^2}{-x^3 + x^2 + 6x}$$

