

### 4.3 What Does it Mean to Be Rational?

#### A Solidify Understanding Task

Part I: Comparing rational numbers and rational fractions.

1. In your own words, define *rational number*. *repeating or terminating decimals or any number that can be written as fraction in simplest form with integers for num. + denom.*  
 Circle the numbers below that are rational and refine your definition, if needed.

$\textcircled{3}$   $\textcircled{-5}$   $\textcircled{\frac{2}{3}}$   $\textcircled{\frac{20}{3}}$   $\textcircled{14}$   $\textcircled{2.7}$   $\sqrt{5}$   $\textcircled{2^3}$   $\textcircled{3^{-3}}$   $\textcircled{\log_2 8}$   $\frac{7}{0}$

2. The definition of a *rational function* is as follows:

A function  $f(x)$  is called a rational function if and only if it can be written in the form

$$f(x) = \frac{P(x)}{Q(x)}$$

where  $P$  and  $Q$  are polynomials in  $x$  and  $Q$  is not the zero polynomial.

Interpret this meaning in your own words and then write three examples of rational functions.

$f(x) = \frac{x+1}{x+2}$  ,  $g(x) = \frac{(x+1)(x-2)}{(x+3)(x-2)}$  ,  $h(x) = \frac{2}{x^2+6x+8}$

3. How are rational numbers and rational functions similar? Different?

S - They can be written as a fraction.  
 D - Rational functions have variables + rational numbers do not.

Part II: Arithmetic of Rational Expressions: making connections between rational numbers and rational expressions. Solve problems in the first column and then use the same process to simplify the rational expressions in the second column.

Arithmetic of rational numbers	Arithmetic of rational expressions
4a. $\frac{2}{3} + \frac{4}{7}$  $\frac{14}{21} + \frac{12}{21} = \frac{28}{21} = \frac{4}{3}$	4b. $\frac{(x-1)^3}{(x+1)} + \frac{4}{(x-1)}$  $\frac{3x-3}{(x+1)(x-1)} + \frac{4x+4}{(x+1)(x-1)} = \frac{7x+1}{(x+1)(x-1)}$
5a. $\frac{3}{8} + \frac{5}{6}$  $\frac{9}{24} + \frac{20}{24} = \frac{29}{24}$	5b. $\frac{(x-1)^2 x}{(x+3)} + \frac{4x}{(x-1)(x+3)}$  $\frac{2x^2-2x}{(x+3)(x-1)} + \frac{4}{(x-1)(x+3)} = \frac{2x^2-2x+4}{(x-1)(x+3)}$ $= \frac{2(x^2-x+2)}{(x-1)(x+3)}$

$= \frac{2(x^2-x+2)}{(x-1)(x+3)}$   
 $= \frac{2(x-2)(x+1)}{(x-1)(x+3)}$

<p>6a. <math>\frac{7}{8} - \frac{1}{6}</math></p> $\frac{21}{24} - \frac{4}{24} = \frac{17}{24}$	<p>6b. <math>\frac{2x}{(x+3)} - \frac{4(x+5)}{(x-1)}</math></p> $\frac{2x^2 - 2x}{(x+3)(x-1)} - \frac{(4x+12)}{(x+3)(x-1)} = \frac{2x^2 - 6x - 12}{(x+3)(x-1)}$
<p>7a. <math>\frac{3}{8} \times \frac{5}{2} = \frac{5}{16}</math></p>	<p>7b. <math>\frac{(x+1)(x-2)}{(x+2)} \times \frac{(x+5)}{(x-2)(x+2)}</math></p> $\frac{(x+1)(x+5)}{(x+2)(x+2)} \text{ or } \frac{(x+1)(x+5)}{(x+2)^2} \text{ or } \frac{x^2 + 6x + 5}{x^2 + 4x + 4}$
<p>8a. <math>\frac{3}{8} \div \frac{5}{6} = \frac{9}{20}</math></p>	<p>8b. <math>\frac{(x+1)(x-2)}{(x+2)} \div \frac{(x+5)}{(x-2)(x+2)}</math></p> $\frac{(x+1)(x-2)}{(x+2)} \times \frac{(x-2)(x+2)}{(x+5)} = \frac{(x+1)(x-2)^2}{x+5}$

or

$$= \frac{(x+1)(x^2 - 4x + 4)}{x+5}$$

$$= \frac{x^3 - 4x^2 + 4x + 4}{x+5}$$

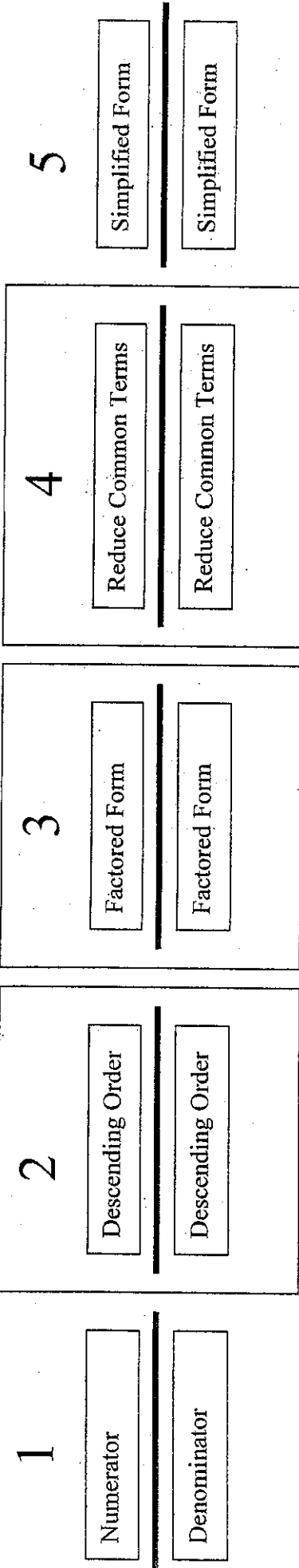
$$= \frac{x^3 - 3x^2 + 4}{x+5}$$

9. To summarize, explain how you would perform the following arithmetic operations on rational expressions:

- Adding: Create equivalent fractions and get a common denominator. Add numerators & keep the common denominator.
- Subtracting: Set a common denominator & subtract N & keep same D.
- Multiplying: Cancel any common factors in the num. & den. Multiply N x N and D x D.
- Dividing: Rewrite division as multiplying by the reciprocal of the second fraction. Cancel any common factors in the N & D. Multiply N x N and D x D. Cancel any common factors.

A rational expression is a quotient of two monomials or polynomials.

# Simplifying Rational Expressions



$x^2 + x^3 + 5 + x^4 + x$   
 Changes to  
 $x^4 + x^3 + x^2 + x + 5$

1. Factor out -1 if the first term is negative.
2. Factor out GCF
3. Factor using difference of squares, diamond, or box etc.

- Reduce GCF's  
 - Reduce pairs of parentheses  
 - If the entire numerator or denominator reduces write 1 in its place

1	2	3 and 4	5
$\frac{13x + 2x^2 + 20}{30 + 2x^2 + 17x}$	$\frac{2x^2 + 13x + 20}{2x^2 + 17x + 30}$	$\frac{(2x + 5)(x + 4)}{(2x + 5)(x + 6)}$	$\frac{x + 4}{x + 6}$
$\frac{10x^2 + 20x + 10}{5x^2 - 5}$	$\frac{10x^2 + 20x + 10}{5x^2 - 5}$	$\frac{10(x + 1)(x + 1)}{5(x + 1)(x - 1)}$	$\frac{2(x + 1)}{(x - 1)}$
$\frac{15 + x - 6x^2}{3x^2 + 4x - 15}$	$\frac{-6x^2 + x + 15}{3x^2 + 4x - 15}$	$\frac{-1(3x - 5)(2x + 3)}{(3x - 5)(x + 3)}$	$\frac{-1(2x + 3)}{(x + 3)}$

\*ODDS together

CC3  
Simplifying Rationals

Simplify.

$$1. \frac{14x^5y^7}{28y^8} = \frac{x^5}{2y}$$

$$2. \frac{12x^3y^7}{3x^4y^6} = \frac{4y}{x}$$

$$3. \frac{4x^2(x+2)}{2(x+2)} = 2x^2$$

$$4. \frac{8x^6(3x-21)}{16x^7(x^2-49)} = \frac{3}{2x(x+7)}$$

$$5. \frac{9(x+1)(x-1)}{3x^2y-3xy} = \frac{3xy(x-1)}{3y^2(x+1)}$$

$$6. \frac{6-x}{x-6} = \frac{-1(-6+x)}{x-6} = \frac{-1(x-6)}{x-6} = -1$$

$$7. \frac{3x^2-3}{6x^2+12x+6} = \frac{3(x^2-1)}{6(x^2+2x+1)} = \frac{3(x+1)(x-1)}{6(x+1)(x+1)} = \frac{x-1}{2(x+1)}$$

$$8. \frac{6x^2-x-15}{-15-x+6x^2} = \frac{(3x-5)(2x+3)}{(3x-5)(x+3)} = \frac{2x+3}{x+3}$$

$$9. \frac{54-6x-4x^2}{4x^2-81} = \frac{-4x^2-6x+54}{4x^2-81} = \frac{-1(4x^2+6x-54)}{(2x+9)(2x-9)} = \frac{-1(2x+9)(2x-6)}{(2x+9)(2x-9)} = \frac{-2(x-3)}{(2x-9)} \text{ or } \frac{-2x+6}{2x-9}$$

\* Odds together  
\* Evens

CC3  
Multiplying Rational Expressions

Simplify the following expressions.

$$1. \frac{2a^2b \cdot 24b^2c}{12a \cdot 15a^2b^2} = \frac{2ac}{3}$$

$$2. \frac{x^2+2x}{4x+8} \cdot \frac{x(x+2)}{3} = \frac{x}{12}$$

$$3. \frac{x^2-y^2}{5x^3y^2} \cdot \frac{4x+4y}{15x^2y^5} = \frac{(x+y)(x-y)}{5x^3y^2} \cdot \frac{4(x+y)}{3 \cdot 5x^2y^5} = \frac{3y^3(x-y)}{4x}$$

$$4. \frac{3p-21}{p^2-49} \cdot \frac{p^2+7p}{3p} = \frac{3(p-7)}{(p+7)(p-7)} \cdot \frac{p(p+7)}{3p} = 1$$

$$5. \frac{x^2-11x+24}{x^2-18x+80} \cdot \frac{x^2-15x+50}{x^2-9x+20} = \frac{(x-8)(x-3)}{(x-10)(x-8)} \cdot \frac{(x-10)(x-5)}{(x-4)(x-5)} = \frac{x-3}{x-4}$$

$$6. \frac{3x^2+10x-8}{3x^2-17x+10} \cdot \frac{x^2+3x-4}{-2x^2+9x+5} = \frac{(3x-2)(x+4)}{(3x-2)(x-5)} \cdot \frac{-1(2x+1)(x-5)}{(x+4)(x-1)} = \frac{-1(2x+1)}{x-1}$$

$$7. \frac{r^2+2r-8}{r^2+4r+3} \cdot \frac{r-2}{3r+3} = \frac{(r+4)(r-2)}{(r+3)(r+1)} \cdot \frac{3(r+1)}{(r-2)} = \frac{3(r+4)}{r+3}$$

Change to multiply

$$8. \frac{x^2+4x-32}{x^2-12x+35} \cdot \frac{x^2-4x-21}{-4x^2+16x} \cdot \frac{x(x-10)}{x^2+11x+24} = \frac{(x+8)(x-4)}{(x-7)(x-5)} \cdot \frac{(x-7)(x+3)}{-4x(x-4)} \cdot \frac{x(x-10)}{(x+8)(x+3)} = \frac{x(x-10)}{-4x(x-5)} = \frac{x-10}{-4(x-5)}$$

$$9. \frac{2x+6}{x^2+x-2} \cdot \frac{x^2+3x+2}{x^2+3x+2} = \frac{2(x+3)}{(x-1)(x+2)} \cdot \frac{(x+2)(x+1)}{(x+3)} = \frac{2(x+1)}{(x-1)}$$

$$10. \frac{t(t^2-t-2)}{t^3-t^2-2t} \cdot \frac{t^2-3t+2}{t^2+3t+2} = \frac{t(t-2)(t+1)}{(t-2)(t-1)} \cdot \frac{t(t+2)}{(t+2)(t+1)} = \frac{t^2}{t-1}$$

CC3  
Rational Expressions

A. Simplify

1.  $\frac{15x^4}{5x^4} = 3$
2.  $\frac{7w^3}{28w^2} = \frac{w}{4}$
3.  $\frac{16p^3}{24p^7} = \frac{2}{3p^4}$
4.  $\frac{9(x-3)}{x-3} = 9$
5.  $\frac{12x-15}{21} = \frac{3(4x-5)}{21} = \frac{4x-5}{7}$
6.  $\frac{4(x-3)}{4x-12} \cdot \frac{4(x+3)}{4(x+3)} = \frac{x-3}{x+3}$
7.  $\frac{(x-4)(x+4)}{x^2-16} \cdot \frac{(x-4)(x+4)}{(x-4)(x+4)} = \frac{x+4}{x-4}$
8.  $\frac{(x-1)(x-8)}{x^2-9x+8} \cdot \frac{(x-1)(x+4)}{(x-1)(x+4)} = \frac{x-8}{x+4}$
9.  $\frac{(x-y)}{x^2-y^2} = \frac{1}{x+y}$

B. Multiply and Simplify

10.  $\frac{x^4}{3x+6} \cdot \frac{5(x+2)}{5x^7} = \frac{1}{3x^3}$
11.  $\frac{(x+4)(x-4)}{x^2-16} \cdot \frac{x+3}{2x+6} = \frac{x+4}{2}$
12.  $\frac{(x+5)(x+5)}{x^2+10x+25} \cdot \frac{x^2-3x}{(x+3)(x-3)} = \frac{x(x+5)}{x+3}$
13.  $\frac{(x-y)(x-y)}{4x+4y} \cdot \frac{x+y}{x+y} = \frac{x+y}{4}$
14.  $\frac{(x-1)(x-1)}{x^2-10x+9} \cdot \frac{x+4}{x^2-5x-36} = \frac{1}{x+1}$
15.  $\frac{(x-3)(x^2+3x+y^2)}{(x+3)(x-3)} \cdot \frac{(x-3)(x-3)}{x^2-6x+9} = \frac{(x-3)^2(x^2+3x+y^2)}{(x+3)(x^2+3x+9)}$

C. Divide and Simplify

16.  $\frac{12x^8}{3x^4} \div \frac{16x^3}{6x} = \frac{3x^9}{2x^2} = \frac{3}{2}x^7$
17.  $\frac{3(x+5)}{3x+15} \div \frac{x+5}{x} = 3$
18.  $\frac{(x+3)(x-3)}{x^2-9} \div \frac{x+2}{x+2} = \frac{(x-3)(x+2)}{x}$
19.  $\frac{(x+4)(x-4)}{x^2-16} \div \frac{(x-5)(x+2)}{3x-12} = \frac{(x+4)(x+2)}{3(x-5)}$
20.  $\frac{(x-5)(x+2)}{x^2-3x-10} \div \frac{(2x-1)(x-3)}{2x^2-11x+5} = \frac{x+2}{x-2}$

Simplify, Multiply and Divide Rational Expressions

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1. Simplify:  $\frac{x^2 - 5x - 14}{x^2 - 49}$   $\frac{(x-7)(x+2)}{(x+7)(x-7)}$   
 A.  $\frac{x+2}{x+7}$  B.  $\frac{2}{7}$  C.  $\frac{x+2}{7-x}$  D.  $\frac{x-2}{x-7}$

2. Simplify:  $\frac{\frac{7x^2y}{21x^2-6x}}{\frac{14x}{49x^2-49}} \div \frac{14x}{7x^2y}$   
 $\frac{7x^2y}{3x(7x-2)} \cdot \frac{7x^2y}{7x^2y} \div \frac{14x}{(7x+2)(7x-2)}$   
 A.  $\frac{6}{7xy+2y}$  B.  $\frac{7xy+2xy}{6}$  C.  $\frac{6}{7xy+2x}$  D.  $\frac{7xy+2y}{6}$

3. Simplify:  $\frac{4(a-7)}{4a-28} + \frac{(a+7)(a-7)}{a^2-49} = \frac{4(a-7)}{3(a-7)} = \frac{4}{3}$   
 A.  $\frac{2(a-7)}{(a+7)}$  B.  $\frac{2a-14}{7}$  C.  $\frac{a^2+4a-77}{6(3a-21)}$  D.  $\frac{a-8}{6(a-7)}$

4. Simplify:  $\frac{3x(x-2)}{3x^2-6x} \cdot \frac{(3x-1)(x+2)}{3x^2+5x-2}$   
 $\frac{3x(x-2) \cdot (3x-1)(x+2)}{-1(x^2-4) \cdot 3(9x^2-1)} = \frac{3x(x-2) \cdot (3x-1)(x+2)}{-1(x+2)(x-2) \cdot 3(3x+1)(3x-1)}$   
 A.  $\frac{x}{-1(3x+1)}$  B.  $\frac{x(x-2)}{(3x-1)(x+2)}$  C.  $\frac{x(x-2)}{(3x-1)(x+2)}$  D.  $\frac{-x(x+2)}{(3x-1)(x+2)}$

5. Multiply:  $\frac{x^2+3x-10}{x^2-4} \cdot \frac{x+2}{x^2-9}$   
 $\frac{(x-2)(x+5)}{(x+2)(x-2)} \cdot \frac{(x+2)}{(x+3)(x-3)}$   
 A.  $\frac{5}{x-9}$  B.  $\frac{x+5}{x-9}$  C.  $\frac{x+5}{x^2-9}$  D.  $\frac{x-5}{x^2+9}$

6. Simplify:  $\frac{2x+6}{\frac{x+1}{x+3} \cdot \frac{(x+1)(x-1)}{(x+3)}}$   
 $\frac{2(x+3)}{x+3} \cdot \frac{(x+1)(x-1)}{(x+3)}$   
 $\frac{2(x-1)}{x+3}$   
 A.  $2(x+1)$  B.  $2(x-1)$  C.  $\frac{2(x+3)^2}{(x+1)^2(x-1)}$  D.  $x+1$

7. Simplify:  $\frac{c^2-3c-10}{(c+2)(c-2)} \div \frac{(c+5)(c-3)}{(c+5)(c-3)}$   
 $\frac{(c-5)(c+2)}{(c+2)(c-2)} \div \frac{(c+5)(c-3)}{(c+5)(c-3)} = \frac{c-5}{c-2}$   
 A.  $\frac{c+2}{2(c-3)}$  B.  $\frac{c-3}{c^2}$  C.  $\frac{c-3}{c-2}$  D.  $\frac{c^2(c-3)}{c-2}$

8. Simplify:  $\frac{2x^2+9x+9}{x^2-6x-27} = \frac{(2x+3)(x+3)}{(x-9)(x+3)}$   
 A.  $\frac{2}{x-3}$  B.  $\frac{1}{3}$  C.  $\frac{2x+3}{x-9}$  D. does not simplify

9. Simplify:  $\frac{x^2-x-6}{x^2-5x+6} = \frac{(x-3)(x+2)}{(x-3)(x-2)}$   
 A. -3 B.  $\frac{x+2}{x-2}$  C.  $\frac{x+3}{x+2}$  D.  $\frac{x-2}{x+2}$

10. Simplify:  $\frac{x^2-x-2}{x^2+6x+5} \cdot \frac{2x^2-3x-2}{4x^2-1}$   
 $\frac{(x+1)(x-2)}{(x+5)(x+1)} \cdot \frac{(2x+1)(2x-1)}{(2x+1)(x-2)}$   
 $\frac{2x-1}{x+5}$

CC3

Adding/Subtracting Rational Expressions

Together

$$1. \frac{2x+5}{5x+10} + \frac{x+1}{5x+10} = \frac{3x+6}{5x+10} = \frac{3(x+2)}{5(x+2)} = \frac{3}{5}$$

To go

$$2. \frac{7a-10}{3a-12} - \frac{a+8}{3a-12} = \frac{7a-10-(a+8)}{3a-12} = \frac{7a-10-a-8}{3a-12} = \frac{6a-18}{3(a-4)} = \frac{2(a-3)}{a-4}$$

Alone

$$3. \frac{6n+5}{2n+6} - \frac{2n-7}{2n+6} = \frac{6n+5-2n+7}{2n+6} = \frac{4n+12}{2n+6} = \frac{4(n+3)}{2(n+3)} = 2$$

$$4. \frac{2x^2-55}{x^2+7x} + \frac{6-x^2}{x^2+7x} = \frac{x^2-49}{x^2+7x} = \frac{(x+7)(x-7)}{x(x+7)} = \frac{x-7}{x}$$

$$5. \frac{7a-15}{a^2-36} - \frac{2a+15}{a^2-36} = \frac{5a-30}{a^2-36} = \frac{5(a-6)}{(a+6)(a-6)} = \frac{5}{a+6}$$

$$6. \frac{5c^2-8c}{c^2-9} + \frac{4c+9c^2}{c^2-9} = \frac{14c^2-4c}{c^2-9} = \frac{2c(7c-2)}{(c+3)(c-3)}$$

Alone

$$7. \frac{y^2-8y}{y^2+10y+16} + \frac{3y-14}{y^2+10y+16} = \frac{y^2-5y-14}{y^2+10y+16} = \frac{(y-7)(y+2)}{(y+2)(y+8)} = \frac{y-7}{y+8}$$

$$8. \frac{3n^2+11n}{n^2+16n+64} - \frac{n^2+40}{n^2+16n+64} = \frac{2n^2+11n+40}{n^2+16n+64} = \frac{(2n-5)(n+8)}{(n+8)(n+8)} = \frac{2n-5}{n+8}$$

Alone

$$9. \frac{4a^2-16}{a^2-8a+16} - \frac{a^2+8a}{a^2-8a+16} = \frac{4a^2-16-a^2-8a}{a^2-8a+16} = \frac{3a^2-8a-16}{(a-4)(a-4)} = \frac{3a+4}{a-4}$$

$$10. \frac{3x+7}{2x^2+10x+12} + \frac{x+5}{2x^2+10x+12} = \frac{4x+12}{2(x^2+5x+6)} = \frac{4(x+3)}{2(x+3)(x+2)} = \frac{2}{x+2}$$



CC3  
Add/Subtract Rational Expressions

- Steps:
1. Factor denominator
  2. Find LCD
  3. Multiply each fraction to get LCD
  4. + or -

LCD:  $9x^3y^3$

$$1. \frac{8 \cdot 3y^2}{3x^3y} + \frac{4}{9xy^3} \cdot \frac{x^2}{x^2} = \frac{24y^2 + 4x^2}{9x^3y^3}$$

LCD:  $4(y+1)$

$$2. \frac{3y+1}{4y+4} - \frac{2y+7}{2y+2} = \frac{1(3y+1) - 2(2y+7)}{4(y+1)}$$

$$= \frac{3y+1-4y-14}{4(y+1)} = \frac{-y-13}{4(y+1)} \text{ or } \frac{-1(y+13)}{4(y+1)}$$

LCD:  $10(x-2)$

$$3. \frac{x+2}{5x-10} - \frac{3x+5}{2x-4} \cdot \frac{5}{5} = \frac{2(x+2) - 5(3x+5)}{10(x-2)}$$

$$= \frac{2x+4-15x-25}{10(x-2)} = \frac{-13x-21}{10(x-2)}$$

LCD:  $15(y+5)$

$$4. \frac{7}{5y+25} - \frac{4}{3y+15} \cdot \frac{5}{5} = \frac{7 - 4}{15(y+5)} = \frac{3}{15(y+5)} = \frac{1}{5(y+5)}$$

LCD:  $3x(x+3)$

$$5. \frac{x}{3x+9} + \frac{8}{x^2+3x} \cdot \frac{3}{3} = \frac{x^2+24}{3x(x+3)}$$

LCD:  $24(y+2)$

$$6. \frac{5x \cdot y}{2y+4} - \frac{6}{y^2+2y} \cdot \frac{2}{2} = \frac{5xy - 12}{24(y+2)}$$

LCD:  $(x+3)(x-3)$

$$7. \frac{4}{x^2-9} + \frac{7}{x+3} \cdot \frac{(x-3)}{(x-3)} = \frac{4}{(x+3)(x-3)} + \frac{7(x-3)}{(x+3)(x-3)} = \frac{4+7x-21}{(x+3)(x-3)} = \frac{-17+7x}{(x+3)(x-3)}$$

LCD:  $(x+6)(x-6)(x-1)$

$$8. \frac{(x-1)}{x^2-36} + \frac{9(x-6)}{x^2+5x-6} = \frac{5x-5+9x-54}{(x+6)(x-6)(x-1)} = \frac{14x-59}{(x+6)(x-6)(x-1)}$$

LCD:  $(x-2)(x+1)(x-1)$

$$9. \frac{2x(x-1)}{x^2-x-2} - \frac{5x(x+1)}{x^2-3x+2} = \frac{2x(x-1) - 5x(x+1)}{(x-2)(x+1)(x-1)} = \frac{2x^2-2x-5x^2-5x}{(x-2)(x+1)(x-1)} = \frac{-3x^2-7x}{(x-2)(x+1)(x-1)} = \frac{-x(3x+7)}{(x-2)(x+1)(x-1)}$$

LCD:  $(y+3)(y-3)(y+2)$

$$10. \frac{y \cdot (y+2)}{y^2-y-6} - \frac{y+2 \cdot (y-3)}{y^2+5y+6} = \frac{y(y+2) - (y+2)(y-3)}{(y-3)(y+2)(y+3)(y+2)} = \frac{y^2+2y - (y^2-y-6)}{(y-3)(y+2)(y+3)(y+2)} = \frac{3y+6}{(y-3)(y+2)(y+3)(y+2)} = \frac{3(y+2)}{(y-3)(y+2)(y+3)(y+2)} = \frac{3}{(y-3)(y+3)(y+2)}$$

LCD:  $x^2-2$

$$11. 3x + \frac{x^2+5x}{x^2-2} = \frac{3x^2+x^2+5x}{x^2-2}$$

LCD:  $24(y-7)$

$$12. \frac{5y}{y^2-7y} - \frac{2y-14}{2(y-7)} + \frac{y}{y} = \frac{3(4y-21)}{y(y-7)}$$

# Answers p 9

$$\textcircled{7} \quad \frac{4}{(x+3)(x-3)} + \frac{7(x-3)}{(x+3)(x-3)} = \frac{4+7x-21}{(x+3)(x-3)} = \frac{7x-17}{(x+3)(x-3)}$$

$$\textcircled{8} \quad \frac{5(x-1)}{(x+6)(x-6)(x-1)} + \frac{9(x-6)}{(x+6)(x-1)(x-6)} = \frac{5x-5+9x-54}{(x+6)(x-6)(x-1)} = \frac{14x-59}{(x+6)(x-6)(x-1)}$$

$$\textcircled{9} \quad \frac{2x(x-1)}{(x-2)(x+1)(x-1)} - \frac{5x(x+1)}{(x-2)(x-1)(x+1)} = \frac{2x^2-2x-5x^2-5x}{(x-2)(x+1)(x-1)}$$

$$= \frac{-3x^2-7x}{(x-2)(x+1)(x-1)} = \frac{-1x(3x+7)}{(x-2)(x+1)(x-1)}$$

$$\textcircled{10} \quad \frac{y(y+3)}{(y-3)(y+2)(y+3)} - \frac{(y+2)(y-3)}{(y+3)(y+2)(y-3)} = \frac{y^2+3y - (y^2-y-6)}{(y+3)(y+2)(y-3)}$$

$$y^2+3y - y^2 + y + 6 \rightarrow 4y+6 \rightarrow 2(y+3)$$

$$\frac{2(y+3)}{(y+3)(y+2)(y-3)} = \frac{2}{(y+2)(y-3)}$$

$$\textcircled{11} \quad \frac{3x(x^2-2)}{1(x^2-2)} + \frac{x^2+5x}{(x^2-2)} = \frac{3x^3-6x+x^2+5x}{x^2-2} = \frac{3x^3+x^2-x}{x^2-2} = \frac{x(3x^2+x-1)}{x^2-2}$$

$$\textcircled{12} \quad \frac{5y \cdot 2}{y(y-7) \cdot 2} - \frac{4 \cdot y}{2(y-7)y} + \frac{9 \cdot 2(y-7)}{y \cdot 2(y-7)} =$$

$$= \frac{10y-4y+18y-126}{2y(y-7)} = \frac{24y-126}{2y(y-7)} = \frac{2(12y-63)}{2y(y-7)} = \frac{12y-63}{y(y-7)}$$

$$= \frac{3(4y-21)}{y(y-7)}$$

CC3  
Rational Expressions

Simplify the following expressions

$$1. \frac{4}{3a} + \frac{8}{3a} = \frac{12}{3a} = \frac{4}{a}$$

$$2. \frac{3}{4a^2b} - \frac{7}{4a^2b} = \frac{-4}{4a^2b} = \frac{-1}{a^2b}$$

$$9. \frac{t^2+3-t}{t^4-16} = \frac{t^2-4}{(t^2+4)(t^2-4)} = \frac{1}{t^2+4}$$

$$4. \frac{4y+2}{y-2} - \frac{y-3}{y-2}$$

$$\frac{4y+2-y+3}{y-2} = \frac{3y+5}{y-2}$$

$$5. \frac{3a-2}{a^2-25} - \frac{4a-7}{a^2-25}$$

$$\frac{3a-2-4a+7}{(a^2-25)} = \frac{-a+5}{(a+5)(a-5)} = \frac{-(a-5)}{(a+5)(a-5)} = \frac{-1}{a+5}$$

$$6. \frac{2a-5}{a^2-9} - \frac{3a-8}{a^2-9} = \frac{2a-5-3a+8}{(a+3)(a-3)} = \frac{-a+3}{(a+3)(a-3)} = \frac{-1(a-3)}{(a+3)(a-3)} = \frac{-1}{a+3}$$

Simplify the following expressions by factoring out a negative 1.

$$7. \frac{a^2}{a-b} + \frac{b^2}{b-a} = \frac{a^2+b^2}{(a-b)(-1)} = \frac{a^2+b^2}{-(a-b)} = \frac{a^2+b^2}{b-a}$$

$$8. \frac{x-7}{x^2-16} + \frac{x-1}{16-x^2} = \frac{x-7}{(x+4)(x-4)} + \frac{x-1}{-(x^2-16)} = \frac{x-7-x+1}{(x+4)(x-4)} = \frac{-6}{(x+4)(x-4)}$$

$$9. \frac{t^2+3}{t^4-16} + \frac{7}{16-t^4} = \frac{t^2+3}{(t^2-4)(t^2+4)} + \frac{7(-1)}{(t^2+4)(t^2-4)} = \frac{t^2+3-7}{(t^2+4)(t^2-4)} = \frac{t^2-4}{(t^2+4)(t^2-4)} = \frac{1}{t^2+4}$$

Find common denominators for the following expressions.

$$10. \frac{a+2}{a-4} + \frac{a-2}{a+3} = \frac{(a+3)(a+2) + (a-4)(a-2)}{(a-4)(a+3)} = \frac{a^2+5a+6 + a^2-6a+8}{(a-4)(a+3)}$$

$$11. \frac{a+3}{a-5} + \frac{a-2}{a+4} = \frac{(a+4)(a+3) - (a+2)(a-5)}{(a-5)(a+4)} = \frac{a^2+7a+12 - (a^2-7a+10)}{(a-5)(a+4)} = \frac{14a+2}{(a-5)(a+4)}$$

$$12. \frac{8}{2x^2-7x+5} + \frac{3x+2}{2x^2-x-10} = \frac{3x^2+7x+14}{(2x-5)(x-1)(x+2)}$$

$$13. \frac{x}{x^2+9x+20} - \frac{4}{x^2+7x+12} = \frac{x(a-5) - 4(a+4)}{(a-5)(a+4)} = \frac{x-5}{(x+5)(x+3)}$$

$$14. \frac{2x+1}{x-y} + \frac{5x^2-5xy}{x^2-2xy+y^2} = \frac{7x+1}{x-y}$$

$$15. \frac{x-3}{x^2-16} - \frac{3x-2}{x^2+2x-24} = \frac{-1(2x^2+7x+10)}{(x+6)(x+4)(x-4)}$$

Add and Subtract Rational Expressions

Name: Ken

Date: \_\_\_\_\_

LCD:  
3 · x

1. Simplify:  $\frac{x(2x+5)}{3} - \frac{5}{x} = \frac{2x^2+5x-15}{3x}$

- A.  $\frac{2x^2+5x-15}{3x}$  B.  $\frac{2}{3}$   
 C.  $\frac{7x-15}{3x}$  D.  $\frac{2x}{3-x}$   
 (2x)(x)

4. Add:  $\frac{5}{2x-8} + \frac{3x}{x^2-16}$

- LCD:  $2(x+4)(x-4)$   
 A.  $\frac{11x}{2(x+4)(x-4)}$  B.  $\frac{11x+4}{2(x+4)(x-4)}$   
 C.  $\frac{11x+20}{2(x+4)(x-4)}$  D.  $11x+20$

$\frac{5x+20+6x}{2(x+4)(x-4)} = \frac{11x+20}{2(x+4)(x-4)}$

5. Simplify:  $\frac{6n-3}{n} - \frac{2n+4}{n} = \frac{4n-7}{n}$

- A.  $\frac{4n+1}{n}$  B. 3  
 C.  $4n+1$  D.  $\frac{4n-7}{n}$

LCD:  
(x+3)(x-3)

2. Simplify:  $\frac{(x-3)}{x+3} + \frac{6}{x-3} = \frac{x^2-6x+9+6x+18}{(x+3)(x-3)}$

- A.  $\frac{x^2+27}{x^2-9}$  B.  $\frac{1}{x^2-9}$   
 C.  $\frac{2x-6}{x^2-9}$  D.  $\frac{2x^2-18}{x^2+9}$   
 $= \frac{x^2+27}{(x+3)(x-3)}$   
 $= \frac{x^2+27}{x^2-9}$

6. Simplify:  $\frac{x-3}{x+2} + \frac{5}{x-2} = \frac{x^2-5x+6+5x+10}{(x+2)(x-2)}$

- LCD:  $(x+2)(x-2)$   
 A.  $\frac{x^2+16}{x^2-4}$  B.  $\frac{1}{x-2} = \frac{x^2+16}{x^2-4}$   
 C.  $\frac{x+2}{2x}$  D.  $\frac{x+2}{x^2-4}$

LCD:  
(x-1)(x-2)

3. Simplify:  $\frac{5x}{x-1} - \frac{2x}{x-2} = \frac{5x^2-10x-2x^2+2x}{(x-1)(x-2)}$

- A.  $\frac{3x^2-8x}{(x-1)(x-2)}$  B.  $\frac{x^2-3x+2}{(x-1)(x-2)}$   
 C.  $\frac{3x^2+8x}{(x-1)(x-2)}$  D.  $\frac{3x^2+12x}{(x-1)(x-2)}$   
 $= \frac{3x^2-8x}{(x-1)(x-2)}$

7. Subtract and simplify:  $\frac{2x+8}{x^2+6x+8} - \frac{x+16}{x^2+8x+12}$

- LCD:  $(x+4)(x+2)(x+6)$   
 A.  $\frac{x-8}{-2x-4}$  B.  $\frac{x-4}{(x+2)(x+6)}$   
 C.  $\frac{x-8}{(x+4)(x+6)}$  D.  $\frac{-x-14}{(x+2)(x+6)}$

$\frac{(2x^2+12x+8x+48) - (x^2+16x+4x+144)}{(x+4)(x+2)(x+6)} = \frac{x^2-16}{(x+4)(x+2)(x+6)}$

$\frac{64}{48} = \frac{4}{3}$

Math 3 Unit 7 Quiz Review

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1. Simplify:  $\frac{4x(3x+1)(2x+3)(3x-1)}{12x^2+4x} \cdot \frac{6x^2+7x-3}{9-4x^2}$   
 $\frac{-1(4x^2-9)}{2(9x^2-1)}$

A.  $\frac{-2x(3x+1)}{(2x+3)(3x-1)}$  B.  $\frac{2x}{2x-3}$

C.  $\frac{-2x}{2x-3}$  D.  $\frac{-2x}{2x+3}$

$\frac{4x(3x+1)(2x+3)(3x-1)}{-1(2x+3)(2x-3) \cdot 2(3x+1)(3x-1)}$   
 $\frac{-2x}{-2x}$

2. Simplify:  $\frac{(x-2)5x}{x-1} - \frac{2x(x-1)}{x-2} = \frac{5x^2-10x-2x^2+2x}{(x-1)(x-2)}$

LCD:  $\frac{(x-1)(x-2)}{(x-2)}$

A.  $\frac{3x^2-8x}{(x-1)(x-2)}$  B.  $\frac{x^2-3x+2}{(x-1)(x-2)}$

C.  $\frac{7x^2+12x}{(x-1)(x-2)}$  D.  $\frac{3x^2+12x}{(x-1)(x-2)}$

$= \frac{3x^2-8x}{(x-1)(x-2)}$

3. Add:  $\frac{(x+4)}{2x-8} + \frac{3x-2}{x^2-16} = \frac{5x+20+6x}{2(x+4)(x-4)}$

A.  $\frac{11x}{2(x-4)^2}$  B.  $\frac{11x}{2(x+4)(x-4)}$

C.  $\frac{11x+4}{2(x+4)(x-4)}$  D.  $\frac{11x+20}{2(x+4)(x-4)}$

Express  $\frac{y}{y+3} \div \frac{y+3}{y}$  in simplest form.

$1 = \frac{y+3}{y+3} \div \left( \frac{y}{y+3} \cdot \frac{y+3}{y} \right)$   
 $\frac{y}{y+3} \div \frac{3}{y+3}$   
 $\frac{y}{y+3} \cdot \frac{y+3}{3} = \frac{y}{3}$

5. Multiply:  $\frac{(x-2)(x+5)}{x^2+3x-10} \cdot \frac{x+2}{x^2-9}$   
 $= \frac{(x+5)}{(x+3)(x-3)}$   
 A.  $\frac{5}{x-9}$  B.  $\frac{x+5}{x-9}$   
 C.  $\frac{x+5}{x^2-9}$  D.  $\frac{x-5}{x^2+9}$

6. Simplify:  $\frac{(x+7)(x-3)}{x^2+4x-21} \div \frac{x+7}{x-3}$   
 A. 1 B. -1  
 C.  $\frac{(x-3)^2}{(x+7)^2}$  D.  $\frac{x+7}{x-3}$

$\frac{(x+7)(x-3)}{(x-3)(x-3)} \cdot \frac{(x-3)}{(x+7)} = 1$

7. Subtract and simplify:  $\frac{(x+4)}{2x+8} - \frac{x+16}{x^2+8x+12}$   
 A.  $\frac{x-8}{-2x-4}$  B.  $\frac{x-4}{(x+2)(x+6)}$   
 C.  $\frac{x-8}{(x+4)(x+6)}$  D.  $\frac{-x-14}{(x+2)(x+6)}$

$= \frac{2x^2+12x+8x+48}{(x+4)(x+2)(x+6)} - \frac{x^2+4x+16x+64}{(x+4)(x+2)(x+6)}$   
 $= \frac{x^2-16}{(x+4)(x+2)(x+6)}$

8. Simplify:  $\frac{4(a-7)}{6} \cdot \frac{3(a-7)}{3a-21}$   
 A.  $\frac{2(a-7)}{(a+7)}$  B.  $\frac{2a-14}{7}$

C.  $\frac{a^2+4a-77}{6(3a-21)}$  D.  $\frac{a-8}{6(a-7)}$   
 $\frac{2 \cdot 4(a-7) \cdot 3(a-7)}{3 \cdot 6 \cdot (a-7)(a+7)} = \frac{2(a-7)}{(a+7)}$

Math 3 Unit 7 Quiz Review 11/19/2015

1.  
Answer: ~~A~~ C  
Objective: A.APR.7

2.  
Answer: A  
Objective: A.APR.7

3.  
Answer: D  
Objective: A.APR.7

4.  
Answer:  $\frac{1}{3}y = \frac{5}{3}$   
Objective: A.APR.7

5.  
Answer: C  
Objective: A.APR.7

6.  
Answer: A  
Objective: A.APR.7

7.  
Answer: B  
Objective: A.APR.7

8.  
Answer: A  
Objective: A.APR.7

# \* Monomial Denominators

A rational equation is an equation containing rational expressions.

CC3

## Solving Rational Equations

$$45. \quad 1. \left( \frac{1}{3} + \frac{4}{5} \right) = \frac{x}{8} \cdot 45$$

LCD  
3 · 5 · 3  
= 45

$$15 + 36 = 5x$$

$$\frac{51 = 5x}{5} \quad \text{Circled Answer: } x = \frac{51}{5}$$

$$6x. \quad 4. \left( \frac{1}{x} - \frac{5}{6x} \right) = \frac{2}{3} \cdot 6x$$

$$6 - 5 = 4x$$

$$1 = 4x$$

Circled Answer:  $x = \frac{1}{4}$

$$7. \quad \frac{1}{4x} - \frac{3}{4} = \frac{7}{x}$$

$$1 - 3x = 28$$

$$-3x = 27$$

Circled Answer:  $x = -9$

$$10. \quad \frac{1}{2} - \frac{2}{7} = \frac{3}{2x}$$

$$7x - 4x = 21$$

$$3x = 21$$

Circled Answer:  $x = 7$

$$40. \quad 2. \left( \frac{7}{8} + \frac{2}{5} \right) = \frac{x}{20} \cdot 40$$

LCD  
2<sup>3</sup> · 5  
= 40

$$35 + 16 = 2x$$

$$51 = 2x$$

Circled Answer:  $x = \frac{51}{2}$

$$2x. \quad 5. \left( \frac{3}{2} - \frac{3}{x} \right) = \frac{9}{2x} \cdot 2x$$

$$3x - 6 = 9$$

$$3x = 15$$

Circled Answer:  $x = 5$

$$8. \quad \frac{2}{y} + \frac{1}{2} = \frac{2}{2y}$$

$$4 + y = 2$$

Circled Answer:  $y = -2$

EV:  
 $y \neq 0$

$$3t. \quad 3. \left( \frac{1}{t} + \frac{1}{3} \right) = \frac{8}{3t} \cdot 3t$$

$$3 + t = 8$$

Circled Answer:  $t = 5$

$$8. \quad 6. \left( \frac{1}{3z} + \frac{1}{8} \right) = \frac{4}{z} \cdot 24z$$

$$8 + 3z = 32$$

$$3z = 24$$

Circled Answer:  $z = 8$

$$9. \quad \frac{4}{3n} - \frac{3}{n} = \frac{10}{8}$$

$$4 - 9 = 10n$$

$$\frac{-5 = 10n}{10} \quad \frac{-5}{10} = \frac{10n}{10}$$

Circled Answer:  $-\frac{1}{2} = n$

\* Polynomial Denominators

→ Factor the denominator

→ Multiply both sides by the LCD of both sides

→ Solve for x

→ Check for extraneous roots (values that make the denominator 0)

CC3

Rational Equations

Solve each rational equation.

1.  $\frac{3x}{4} = \frac{5x+1}{3}$

$9x = 4(5x+1)$

$9x = 20x+4$

$-11x = 4$

$x = -\frac{4}{11}$

2.  $\frac{5-\frac{4}{x+1}}{1} = 6 \cdot (x+1)$

$x+1 \neq 0$

$x \neq -1$

$5(x+1) - 4 = 6(x+1)$

$5x+5-4 = 6x+6$

$5x+1 = 6x+6$

$x = -5$

3.  $\frac{x}{x+3} - \frac{x}{x-3} = \frac{x^2+9}{x^2-9}$

$x(x-3) - x(x+3) = x^2+9$

$x^2-3x - x^2-3x = x^2+9$

$-6x = x^2+9$

$x^2+6x+9 = 0$

$(x+3)^2 = 0$

$x+3 = 0$

$x = -3$

Extraneous root

4.  $\frac{7}{x^2-5x} + \frac{2}{x} = \frac{3}{2x-10}$

$14 + 2 \cdot 2(x-5) = 3 \cdot x$

$14 + 4x - 20 = 3x$

$x = 6$

LCM:

$(x+4)$

$(x-1)$

$(x-4)$

EV:

$x \neq -4$

$x \neq 1$

$x \neq 4$

5.  $\frac{x}{x^2+3x-4} = \frac{x}{x^2-16}$

$(x-1)(x+2) = (x+3)(x-4)$

$x^2+x-2 = x^2-x-12$

$2x = -10$

$x = -5$

6.  $\frac{4}{x-3} = \frac{2}{x+1} + \frac{16}{x^2-2x-3}$

$4(x+1) = 2(x-3) + 16$

$4x+4 = 2x-6+16$

$2x+4 = 10$

$2x = 6$   
 $x = 3$

extraneous root

EV:

$x \neq 3, -1$



Rational Equations

Name \_\_\_\_\_

Solve each equation. Write the final answer in a solution set. Show your work.

$$4(x-3) \cdot 1. \left[ \frac{9}{x-3} \right] = \left[ \frac{x-4}{x-3} + \frac{1}{4} \right] 4(x-3)$$

$$36 = 4(x-4) + (x-3)$$

$$36 = 4x - 16 + x - 3$$

$$36 = 5x - 19$$

$$55 = 5x$$

$$x = 11 \quad \{11\}$$

$$(b+2)(b+3) \cdot 2 \left[ \frac{3}{b^2+5b+6} + \frac{b-1}{b+2} \right] = \left( \frac{7}{b+3} \right) (b+2)(b+3)$$

$$3 + (b+3)(b-1) = 7(b+2)$$

$$3 + b^2 + 2b - 3 = 7b + 14$$

$$b^2 - 5b - 14 = 0$$

$$(b-7)(b+2) = 0$$

$$b = 7 \text{ or } -2$$

$\{7\}$   
 extraneous root

$(n+4)(n-2)$

$$3. \left[ \frac{1}{n-2} \right] = \left[ \frac{2n+1}{n^2+2n-8} + \frac{2}{n+4} \right] (n+4)(n-2)$$

$$n+4 = (2n+1) + 2(n-2)$$

$$n+4 = 2n+1+2n-4$$

$$n+4 = 4n-3$$

$$7 = 3n$$

$$n = 7/3$$

$\{7/3\}$

$(x+3)(x-3)$

$$4. \left[ \frac{4x^2}{x^2-9} - \frac{2x}{x+3} \right] = \left( \frac{3}{x-3} \right) (x+3)(x-3)$$

$$4x^2 - 2x(x-3) = 3(x+3)$$

$$4x^2 - 2x^2 + 6x = 3x + 9$$

$$2x^2 + 3x - 9 = 0$$

$$(2x-3)(x+3) = 0$$

$$x = 3/2 \text{ or } -3 \quad \{3/2\}$$

CD:  $(z+2)(z-2)$

$$5. \frac{5z+2}{z^2-4} = \left[ \frac{-5z}{2-z} + \frac{2}{z+2} \right] \cdot \frac{1}{-(z-2)}$$

$$\frac{5z+2}{(z+2)(z-2)} = \frac{5z \cdot z+2}{(z-2) \cdot z+2} + \frac{2 \cdot z-2}{(z+2) \cdot z-2}$$

$$5z+2 = 5z^2+10z+2z-4$$

$$0 = 5z^2+7z-6$$

$$0 = (5z-3)(z+2)$$

$$z = 3/5 \text{ or } -2 \quad \{3/5\}$$

CD:  $4(at+3)$

$$6. \frac{a-4}{a+3} = \frac{3a+2}{a+3} + \frac{a}{4(at+3)}$$

$$4(a-4) = 4(3at+2) + a(at+3)$$

$$4a-16 = 12a+8+a^2+3a$$

$$0 = a^2+11a+24$$

$$0 = (a+8)(a+3)$$

$$a = -8 \text{ or } -3$$

$\{-8\}$

## Graphing Rational Functions

N = Numerator

D = Denominator

H.A. = Horizontal Asymptote

V.A. = Vertical Asymptote

O.A. = Oblique Asymptote (slant, linear)

Before factoring, find:

- Y-intercept: The ratio of the constant terms (N/D)
- H.A.: if degree in N = degree in D, then  $y =$  ratio of coefficients (highest degree terms)
- H.A.: if degree of N < degree of D, then  $y = 0$
- H.A.: if degree of N > degree of D, there is no H.A., but if degree of N is 1 degree higher than D, it is oblique asymptote (O.A.)

The O.A. can be found by dividing the N by the D, using long or synthetic division. When your quotient is a linear equation, you have found the O.A.

After factoring, find:

- Simplify by dividing out any common factors
- Hole: If an expression was divided out, set that expression = to zero and solve for x. the substitute the x value into the expression and solve for y.
- V.A.: The expression left in the denominator should be set = to zero and solved. The answer is the V.A.
- X-intercept: solve the expression left in the numerator by setting it equal to zero.
- Domain: find the values of the D that would make it = zero. Set the expression to zero and solve. The values will be restricted from the domain. The domain will be all real numbers except the excluded values.

# Everything I need to know about Rational Functions

yes finding  $y = \frac{A}{(x-B)} + C$

(2)

Y-intercept  
 where  $x = 0$   
 $(0, 2)$

X-intercept  
 where  $y = 0$   
 $(-2, 0)$

Case 1  
 Original Equation

$$\frac{1x^2 - 4}{2x^2 - 3x - 2}$$

$$\frac{(x+2)(x-2)}{(2x+1)(x-2)}$$

Factored Form

Horizontal Asymptote

Case 1:  
 Degree of Num = Deg of Den  
 easy =  $\frac{\text{leading coeff}}{\text{leading coeff}}$   
 $y = \frac{1}{2}$

Case 2:  
 Deg of Num < Deg of Den  
 easy:  $y = 0$

Case 3:  
 Deg of Num > Deg of Den  
 oblique asymptote  
 see long division!

Vertical Asymptote

if a value is undefined after simplifying, that value will be an asymptote at that value.

$2x + 1 = 0$   
 $x = -\frac{1}{2}$

Form:  $x \neq \#$

Domain

$2x + 1 \neq 0$      $x - 2 \neq 0$   
 $x \neq -\frac{1}{2}$      $x \neq 2$

(all reals except  $-\frac{1}{2}$  and 2)

Hole

occurs whenever factor in numerator & denominator cancel out  
 occurs when the factor = 0  
 $x = 2$

Algebra 2  
Rational Functions

Name: \_\_\_\_\_  
Date: \_\_\_\_\_

①

Case 2

Original Equation:  $\frac{x-1}{2x^2-x-1}$  = Factored Form:  $\frac{(x-1)}{(2x+1)(x-1)}$

Y-intercept: (0,1)

X-intercept: None (all numerator terms reduced)

Horizontal Asymptote:  $y=0$

Vertical Asymptote:  $x=-\frac{1}{2}$

Domain:  $x \neq -\frac{1}{2}, x \neq 1$

Hole:  $x=1$

②

Case 2

Original Equation:  $\frac{x+2}{x^2+5x+6}$  = Factored Form:  $\frac{(x+2)1}{(x+2)(x+3)}$

Y-intercept:  $(0, \frac{1}{3})$

X-intercept: None

Horizontal Asymptote:  $y=0$

Vertical Asymptote:  $x=-3$

Domain:  $x \neq -2 \text{ or } -3$   
 $x \in \mathbb{R}$

Hole:  $x=-2$

$y = \frac{0x/2}{x^2}$

③

Case 2

Original Equation:  $\frac{x-3}{2x^2-5x-3}$  = Factored Form:  $\frac{x-3}{(2x+1)(x-3)}$

Y-intercept: (0,1)

X-intercept: none

Horizontal Asymptote:  $y=0$

Vertical Asymptote:  $x=-\frac{1}{2}$

Domain:  $x \neq -\frac{1}{2}$   
or 3  
 $x \in \mathbb{R}$

Hole:  $x=3$

④

Case 1

Original Equation  $\frac{x^2 + 7x - 8}{x^2 - 3x - 2}$  =  $\frac{(x-8)(x+1)}{(x-2)(x-1)}$  Factored Form

Y-intercept (0, 7)      X-intercept (8, 0) and (-1, 0)

Horizontal Asymptote  $y = 1$

Vertical Asymptote  $x = 2$  and  $x = 1$

Domain  $x \neq 1, x \neq 2$

Hole (No terms reduced)

⑤

Case 1

Original Equation  $\frac{x^2 - 6x + 8}{x^2 + 6x + 8}$  =  $\frac{(x-4)(x-2)}{(x+4)(x+2)}$  Factored Form

Y-intercept (0, 1)      X-intercept (4, 0) and (2, 0)

Horizontal Asymptote  $y = 1$

Vertical Asymptote  $x = -4$  and  $x = -2$

Domain  $x \neq -4$  or  $-2$   
 $x \in \mathbb{R}$

Hole none

⑥

Case 3

Original Equation  $\frac{2x^2 + 3x - 5}{x+2}$  =  $\frac{(2x+5)(x-1)}{x+2}$  Factored Form

Y-intercept  $(0, -\frac{5}{2})$       X-intercept  $(1, 0)$  and  $(-\frac{5}{2}, 0)$

Horizontal Asymptote none

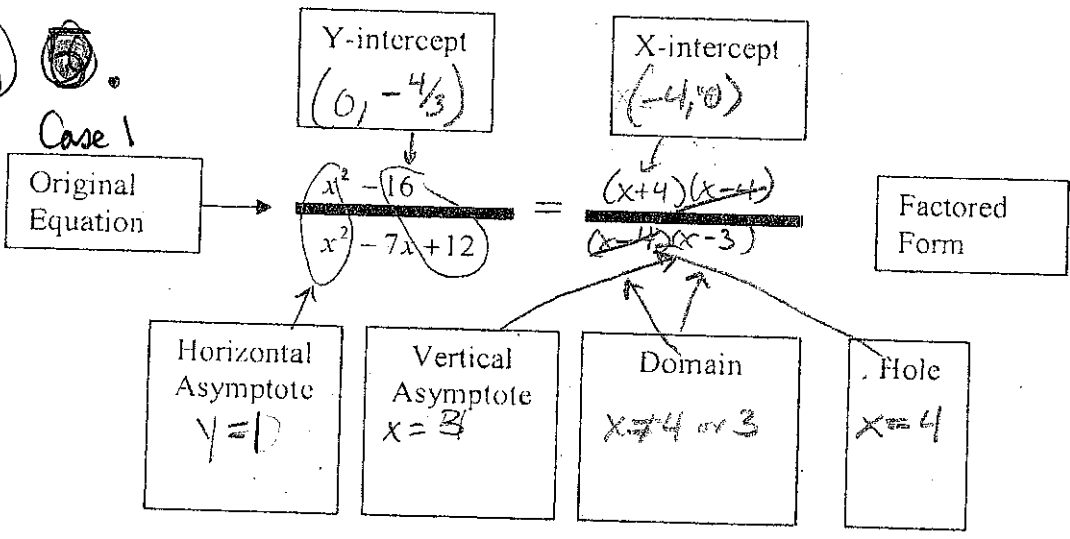
Vertical Asymptote  $x = -2$

Domain  $x \neq -2$   
 $x \in \mathbb{R}$

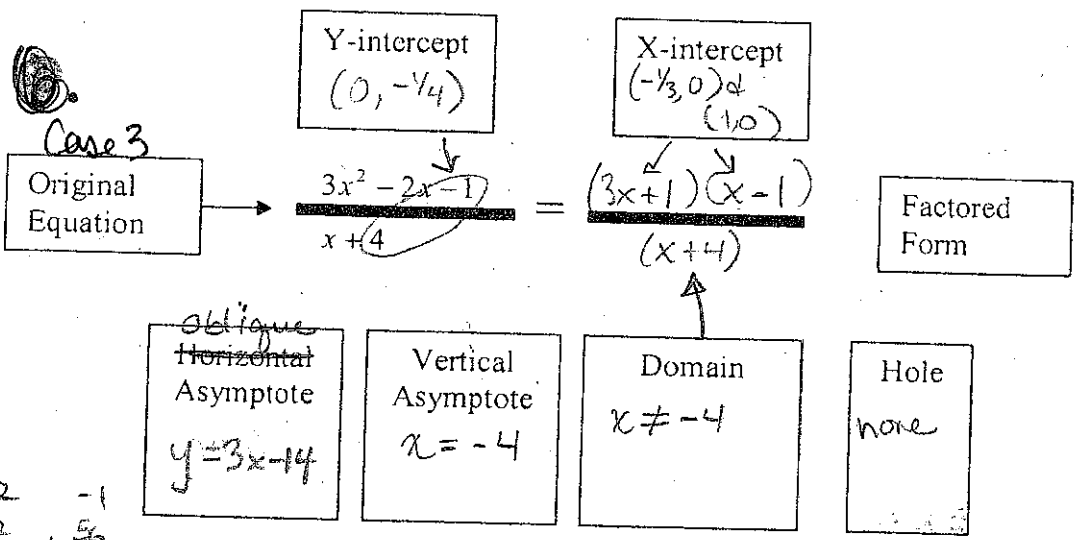
Hole none

$$\begin{array}{r} -2 \overline{) 2} \quad 3 \quad -5 \\ \underline{-4} \phantom{0} \phantom{0} \\ 2 \phantom{0} \phantom{0} \\ \underline{-2} \phantom{0} \phantom{0} \\ 0 \phantom{0} \phantom{0} \end{array}$$

7 ~~8~~  
Case 1



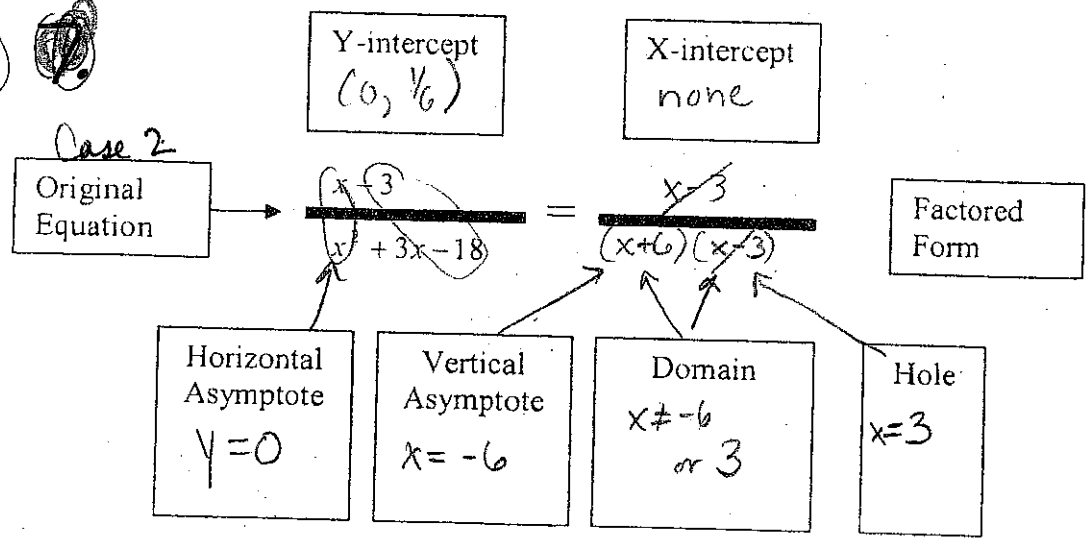
8 ~~7~~  
Case 3



oblique asymptote

$$\begin{array}{r} -4 \overline{) 3} \quad -2 \quad -1 \\ \underline{-12} \phantom{-1} \\ 3 \phantom{-12} \underline{-14} \phantom{-1} \\ \phantom{3} \phantom{-12} \phantom{-14} \underline{56} \phantom{-1} \end{array}$$

9 ~~8~~  
Case 2



Rational Functions

Name: \_\_\_\_\_

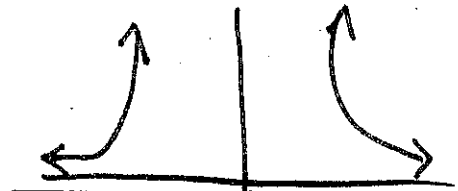
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Case 2

Original Equation  $\rightarrow \frac{0x^3}{x^3 - 4x} = \frac{4x}{x(x+2)(x-2)}$

Y-intercept  
none

X-intercept  
none



Factored Form

Horizontal Asymptote  
 $y=0$

Vertical Asymptote  
 $x=2$   
 $x=-2$

Domain  
 $x \in \mathbb{R}$   
 $x \neq 2, -2$

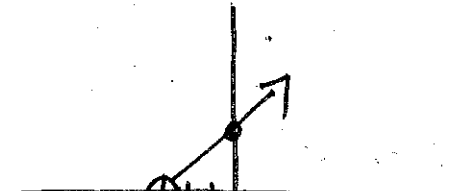
Hole  
 $x=0$

Case 3

Original Equation  $\rightarrow \frac{0x^2}{x^2 + 6x + 9} = \frac{(x+3)(x+3)}{x+3}$

Y-intercept  
(0,3)

X-intercept  
(-3,0)



Factored Form

Horizontal Asymptote  
none

Vertical Asymptote  
none

Domain  
 $x \in \mathbb{R}$   
 $x \neq -3$

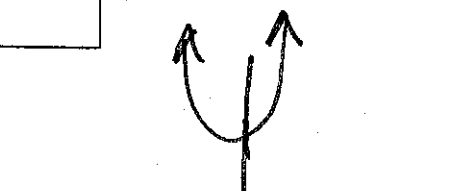
Hole  
 $x=-3$

Case 1

Original Equation  $\rightarrow \frac{y = \frac{100}{15}}{\frac{4(x^2 - 25)}{4x^2 - 100}} = \frac{4(x+5)(x-5)}{(2x-5)(x+3)}$

Y-intercept  
(0, 6 2/3)

X-intercept  
(-5,0)  
(5,0)



Factored Form

Horizontal Asymptote  
 $y=2$

Vertical Asymptote  
 $x = \frac{5}{2}$   
 $x = -3$

Domain  
 $x \in \mathbb{R}$   
 $x \neq \frac{5}{2}, -3$

Hole  
none

# Graphing Rational Expressions

**Example 1:** Simplify the following. State any restrictions on the variables.

a)  $\frac{(x+1)(x-5)}{(x-5)(x^2-1)} = \frac{\cancel{(x+1)}\cancel{(x-5)}}{\cancel{(x-5)}(x+1)(x-1)}$   
 $\frac{1}{x-1}$  restrictions:  $x \neq 5, -1, 1$

b)  $\frac{x^2+x-12}{x^2+7x+12} = \frac{\cancel{(x+4)}(x-3)}{\cancel{(x+4)}(x+3)}$   
 $\frac{x-3}{x+3}$  restrictions:  $x \neq -4, -3$

**Vertical Asymptotes:** Where the denominator of a function equals zero.

**Point of Discontinuity:** A hole in the graph.

→ No division by zero!

$\frac{N}{D}$

\* Anything that cancels creates a hole! ○

**Example 2:** Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of  $f(x) = \frac{x^2-1}{x^2-6x+5}$

$f(x) = \frac{x^2-1}{x^2-6x+5} = \frac{\cancel{(x+1)}\cancel{(x-1)}}{(x-5)(x-1)}$

**Example 3:** Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of  $f(x) = \frac{x^2-4}{x^2+5x+6}$

$f(x) = \frac{x^2-4}{x^2+5x+6} = \frac{\cancel{(x+2)}(x-2)}{(x+3)(x+2)}$

\* Hole at  $x=1$  since  $(x-1)$  cancels out.

\* Vertical asymptote at  $x=+5$  since it doesn't cancel and would cause denominator to equal 0.

\* Hole @  $x=-2$ .

\* VA @  $x=-3$ .

**Horizontal Asymptotes:** determined by comparing the degree of the numerator to the degree of the denominator. Let  $m$  = degree of numerator and  $n$  = degree of denominator.

Then the graph has:	Then the graph has:
<p><math>m &lt; n</math> <b>Case 2</b></p> <p><math>f(x) = \frac{0x^2x+4}{x^2+5x+4} = \frac{x+4}{(x+4)(x+1)}</math></p>	<p>A horizontal asymptote at <math>y=0</math></p> <p>V.A.: <math>x = -1</math> Hole(s): <math>x = -4</math></p> <p>H.A.: <math>y = 0</math> Domain: <math>x \in \mathbb{R}</math> except <math>x \neq -1, -4</math></p>
<p><math>m = n</math> <b>Case 1</b></p> <p><math>f(x) = \frac{x^2+5x+4}{4x^2-9} = \frac{(x+4)(x+1)}{(2x+3)(2x-3)}</math></p>	<p>A horizontal asymptote at the coefficient of <math>m</math> divided by the coefficient of <math>n</math></p> <p>V.A.: <math>x = \frac{-3}{2}, x = \frac{3}{2}</math> Hole(s): <u>none</u></p> <p>H.A.: <math>y = \frac{1}{4}</math> Domain: <math>x \in \mathbb{R}</math> except <math>x \neq</math></p>
<p><math>m &gt; n</math> <b>Case 3</b></p> <p><math>f(x) = \frac{\cancel{(x+4)}\cancel{(x+1)}}{0x^2x+4} = \frac{1}{4}</math>                      ↪ can't ÷ by 0.</p>	<p>No horizontal asymptote</p> <p>V.A.: <u>none</u> Hole(s): <math>x = -4</math></p> <p>H.A.: <u>none</u> Domain: <math>x \in \mathbb{R}</math> except <math>x \neq</math></p>

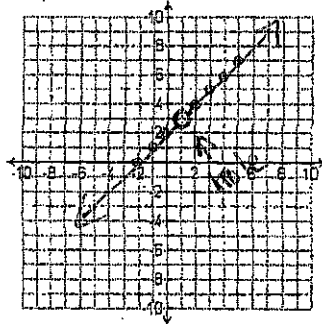


Example 4: State the asymptotes and points of discontinuity of each equation, and then graph the function and state the domain.

a)  $f(x) = \frac{x^2 + x - 2}{0x^2 - x - 1}$

$\frac{(x-1)(x+2)}{x-1}$

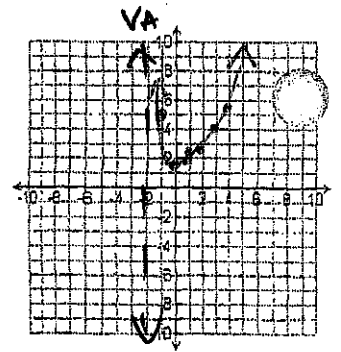
Graph is a line with a hole @  $x=1$ .



VA: none H:  $x=1$

HA: none D:  $x \in \mathbb{R}$   
 $x \neq 1$

b)  $f(x) = \frac{2x^2 + 3}{0x^2 - x + 2}$

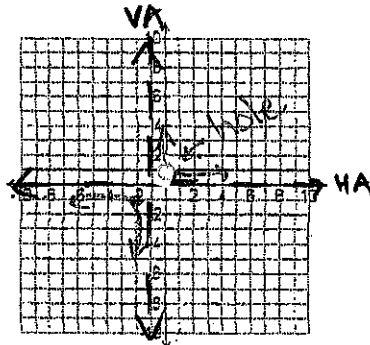


Vertical Asymptote:  $x=-2$  Hole: none

Horizontal Asymptote: none Domain:  $x \in \mathbb{R}$   
 $x \neq -2$

c)  $f(x) = \frac{0x^2 - x - 1}{x^2 - 1}$

$\frac{-x-1}{(x+1)(x-1)}$

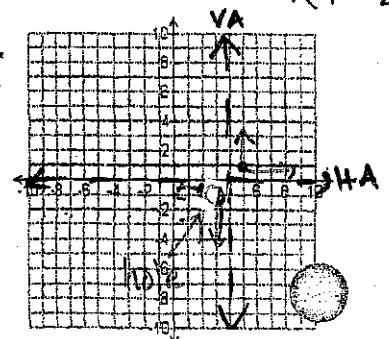


VA:  $x=-1$  H:  $x=1$

HA:  $y=0$  D:  $x \in \mathbb{R}$   
 $x \neq 1, -1$

d)  $f(x) = \frac{0x^2 - x - 3}{x^2 - 7x + 12}$

$\frac{-x-3}{(x-4)(x-3)}$

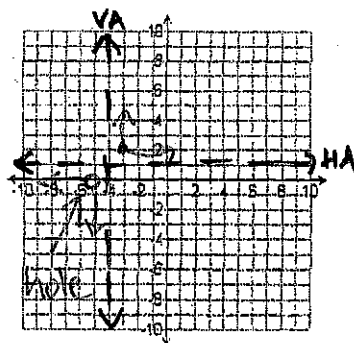


VA:  $x=4$  H:  $x=3$

HA:  $y=0$  D:  $x \in \mathbb{R}$   
 $x \neq 3, 4$

e)  $f(x) = \frac{x^2 + 10x + 25}{x^2 + 9x + 20}$

$\frac{(x+5)(x+5)}{(x+5)(x+4)}$

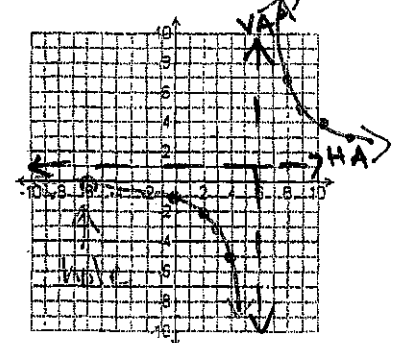


VA:  $x=-4$  H:  $x=-5$

HA:  $y=1$  D:  $x \in \mathbb{R}$   
 $x \neq -5, -4$

f)  $f(x) = \frac{x^2 + 12x + 36}{x^2 - 36}$

$\frac{(x+6)(x+6)}{(x+6)(x-6)}$



VA:  $x=6$  H:  $x=-6$

HA:  $y=1$  D:  $x \in \mathbb{R}$   
 $x \neq -6, 6$  23

Name: Key

Date: \_\_\_\_\_

1. Solve:  $\frac{2(x-7)}{x^2+3x-28} + \frac{x-2}{x-4} = \frac{x+3}{x+7}$

$x \neq 4$   
 $x \neq -7$

$$2(x-7) + (x+7)(x-2) = (x-4)(x+3)$$

$$2x - 14 + x^2 + 5x - 14 = x^2 - x - 12$$

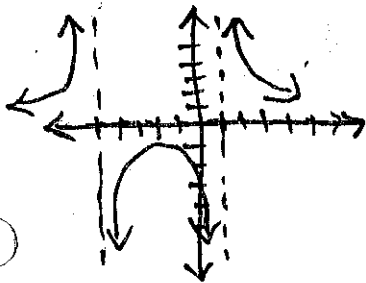
$$7x - 28 = -x - 12 + 28$$

$$8x = 16$$

$$x = 2$$

2. Graph the following. Be sure to label important points.

$$g(x) = \frac{1}{x^2+4x-5} = \frac{1}{(x+5)(x-1)}$$



Yint:  $(0, -1/5)$   
 Xint: none  
 Hole: none  
 Domain:  $x \in \mathbb{R}, x \neq -5, 1$   
 VA:  $x = -5, x = 1$   
 HA:  $y = 0$

3. If Chad can paint 4 rooms in 6 hours and Cierra can do it in 10 hours, then how many hours would it take them working together?

	Chad	Cierra
T	x	x
A	6	10

$$\frac{30}{x} + \frac{30}{x} = 1$$

$$5x + 3x = 30$$

$$8x = 30$$

$$x = 3\frac{3}{8} \text{ hours}$$

4. Paul can plant his wheat crop in 10 days. His daughter can do it in 15 days. How many days will it take if they work together?

- A. 7.5    B. 7    C. 6    D. 8

	Paul	daughter
T	x	x
A	10	15

$$\frac{30}{x} + \frac{30}{x} = 1$$

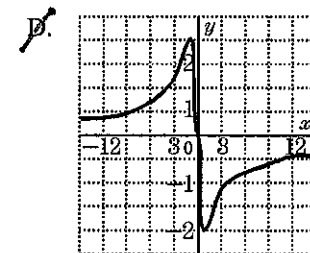
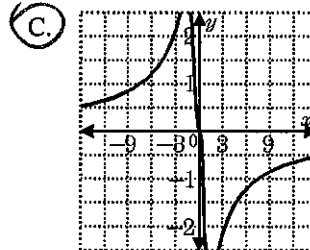
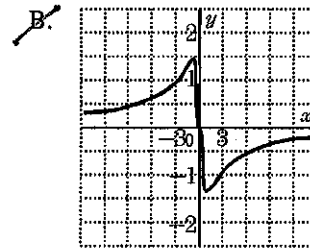
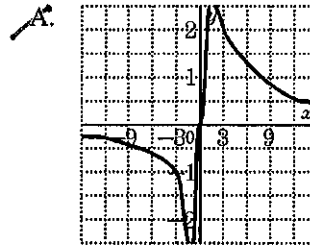
$$3x + 2x = 30$$

$$5x = 30$$

$$x = 6$$

5. Which of the following represents the graph of  $y = \frac{-8x}{x^2+1}$ ?

x	y
3	$-24/10 = -2\frac{2}{5}$
0	0
-3	$24/10 = 2\frac{2}{5}$



6. Solve for x:  $\frac{1}{2x-3} = \frac{5}{4x+1}$

$$4x+1 = 5(2x-3)$$

$$4x+1 = 10x-15$$

$$\frac{16}{6} = \frac{6x}{6}$$

$$2\frac{2}{3} = \frac{8}{3} = x$$

LCM  
 $-1/(x+1)(x-1)$

$x \neq 1, -1$

7. Solve:  $\frac{2}{x^2-1} + \frac{1}{1-x} = 0$   
 $(x+1)(x-1) -1(x-1)$

$\frac{2}{3} + \frac{1}{3} \neq 0$

$-2 + x + 1 = -1(x+1)(x-1)$

no solution

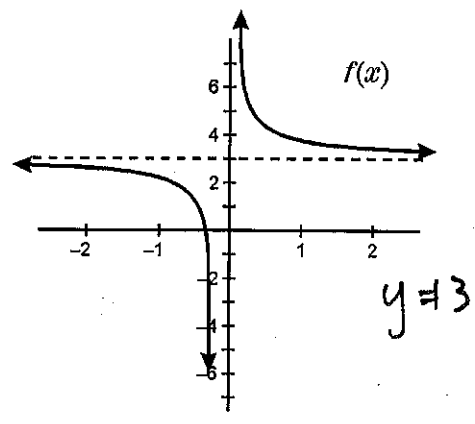
$x - 1 = -x^2 + 1$

$x^2 + x - 2 = 0$

$(x-1)(x+2) = 0$

~~x=1~~  
~~x=2~~

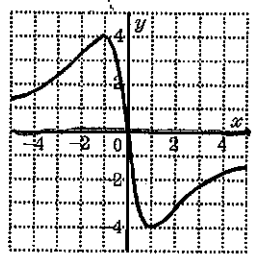
8. What value(s) are restricted from the range of  $f(x)$ ?



- A. 1
- B. 0
- C. 3**
- D. there are no restricted values

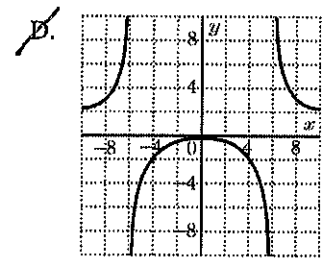
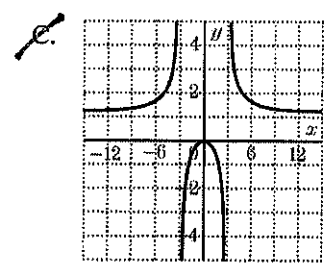
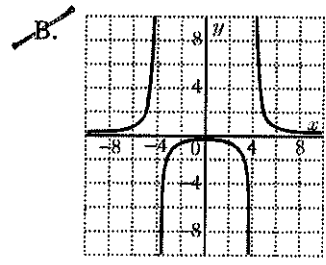
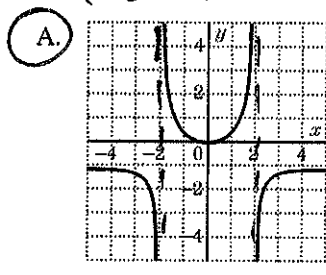
9. Which of the following is the equation of an asymptote for the function graphed?

- A.  $x = 0$
- B.  $y = 0$**
- C.  $x = -4$
- D.  $x = 4$



10. Which of the following represents the graph of

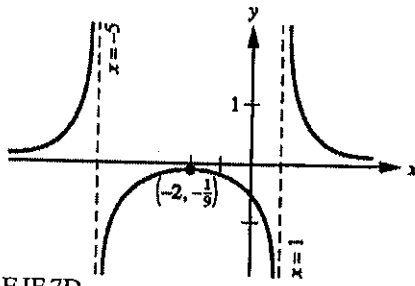
$y = -\frac{x^2}{(x+2)(x-2)}$



Unit 7 Quiz 2 Review 11/30/2015

1.  
 Answer: 2  
 Objective: A.REI.2

2.  
 Answer:



Objective: F.IF.7D

3.  
 Answer: 3.75 hours  
 Objective: Solving applications of rational equations

4.  
 Answer: C

5.  
 Answer: C  
 Objective: F.IF.7D

6.  
 Answer:  $2\frac{2}{3}$   
 Objective: A.REI.2

7.  
 Answer:  $\emptyset$   
 Objective: A.REI.2

8.  
 Answer: C  
 Objective: F.IF.5

9.  
 Answer: B  
 Objective: F.IF.7D

10.  
 Answer: A  
 Objective: F.IF.7D

1.

Y-intercept  
(0, 3)

X-intercept  
(-2, 0) and  
(1, 0)

Original Equation

$$\frac{x^2 + 11x - 12}{x - 4} = \frac{(x+2)(x-6)}{(x-4)}$$

Factored Form

Horizontal Asymptote  
no horizontal

Vertical Asymptote  
 $x = 4$

Domain  
 $\mathbb{R}$   
 $x \neq 4$

Hole  
none

2.

Y-intercept  
none

X-intercept  
(2, 0)

Original Equation

$$\frac{2x^2 - x}{x^2 - 8x} = \frac{x(2x-1)}{x(x-8)}$$

Factored Form

Horizontal Asymptote  
 $y = 2$

Vertical Asymptote  
 $x = 8$

Domain  
 $\mathbb{R}$   
 $x \neq 0$  or  $8$

Hole  
 $x = 0$

$y = \frac{1}{x}$

Function	Transformation	Domain	Vert. Asymp.	Horiz. Asymp.
3. $y = \frac{3}{x+1}$	left 1 vertical stretch by 3	$\mathbb{R}, x \neq -1$	$x = -1$	$y = 0$
4. $y = \frac{-1}{x} + 4$	reflect over x-axis up 4	$\mathbb{R}, x \neq 0$	$x = 0$	$y = 4$

Simplify.

5.  $\frac{x^2 + 2x + 1}{2x^2 + 3x + 1} = \frac{x+1}{2x+1}$

$= \frac{(x+1)(x+1)}{(2x+1)(x+1)}$

6.  $\frac{(x+2)(x-2)}{x^2 - 4x + 4} \cdot \frac{2x-1}{x+2} = \frac{2x-1}{x-2}$

Assignment

Simplify each expression.

$$1) \frac{5(7p+6)}{35p+30} \div \frac{3(7p+6)}{21p+18}$$

$$\frac{5}{5} \cdot \frac{1}{3} = \frac{1}{3}$$

$$3) \frac{6n^3 - 48n^2}{6n^2} \div \frac{n^2 - 16n + 64}{5n}$$

$$\frac{6n^2(n-8)}{6n^2} \cdot \frac{5n}{(n-8)(n-8)} = \frac{5n}{n-8}$$

$$5) \frac{6x^2 - 18x}{x-7} \cdot \frac{1}{6x}$$

$$\frac{6x(x-3)}{x-7} \cdot \frac{1}{6x} = \frac{x-3}{x-7}$$

$$7) \frac{8x^2 - 16x}{16x - 8x^2} \cdot \frac{x+8}{5}$$

$$\frac{8x(x-2)}{-1(8x)(x-2)} \cdot \frac{x+8}{5} = \frac{-1(x+8)}{5}$$

LCD:  $\frac{(p-4)(p-8)}{(p-4)(p-8)}$

$$\frac{(p-4) \cdot 7}{(p-4)p-8} - \frac{(p-8)}{p-4} = \frac{(7p-28) - (p^2-p-56)}{(p-8)(p-4)}$$

$$= \frac{-p^2 + 8p + 28}{(p-8)(p-4)}$$

LCD:  $\frac{5(x+1)x}{5(x+1)x}$

$$11) \frac{x \cdot 3x}{5x+5} - \frac{4}{x+1}$$

$$= \frac{3x^2 - 4x - 4}{5x(x+1)}$$

$$= \frac{(3x+2)(x-2)}{5x(x+1)}$$

$$13) \frac{8(5a+3)}{7} + \frac{a-6}{40a+24}$$

$$= \frac{448a + 168 + a - 6}{8(5a+3)}$$

$$= \frac{449a + 162}{40a + 24}$$

$$15) \frac{7n}{n^2-9} - \frac{3n-7}{n^2-9}$$

$$= \frac{7n^3 - 63n - 3n + 7}{n^2-9}$$

$$= \frac{7n^3 - 66n + 7}{n^2-9}$$

$$2) \frac{(x+6)(x-4)}{x^2+2x-24} \cdot \frac{1}{x+6}$$

$$= \frac{x-4}{(x+5)(x-1)}$$

$$4) \frac{x^2 - 13x + 42}{x-7} \div \frac{6-x}{7x^2}$$

$$\frac{(x-6)(x-7)}{(x-7)} \cdot \frac{7x^2}{-1(x-6)} = -7x^2$$

$$6) \frac{x+1}{x-4} \div \frac{6}{6x^3-24x^2}$$

$$\frac{x+1}{x-4} \cdot \frac{6x^2(x-4)}{6} = x^2(x+1)$$

or  $x^3 + x^2$

$$8) \frac{6p^3 - 24p^2}{7p} \cdot \frac{p+6}{p^2+2p-24}$$

$$\frac{6p^2(p-4)}{7p} \cdot \frac{(p+6)}{(p-4)(p+6)} = \frac{6p}{7}$$

$$10) \frac{5x^2(4x-3)}{7x+2} - \frac{(3x-5)(x+2)}{5x^2}$$

$$\frac{(20x^3 - 15x^2) - (21x^2 - 29x - 10)}{5x^2(7x+2)} = \frac{20x^3 - 36x^2 + 29x + 10}{5x^2(7x+2)}$$

$$12) \frac{(k-6)}{3k} + \frac{(5k-8)(3k)}{k-6}$$

$$\frac{8k - 48 + 15k^2 - 24k}{3k(k-6)} = \frac{15k^2 - 16k - 48}{3k(k-6)}$$

$$14) \frac{5x^2(x-1)}{7x-7} - \frac{5(7x-7)}{5x^2}$$

$$\frac{5x^3 - 5x^2 - 35x + 35}{5x^2(7x-7)} = \frac{5(x^3 - x^2 - 7x + 7)}{5x^2 \cdot 7(x-1)}$$

$$= \frac{x^2 - 7}{7x^2}$$

$$16) \frac{(3n-5)6}{n-7} + \frac{3n(n-7)}{3n-5}$$

$$\frac{18n - 30 + 3n^2 - 21n}{(n-7)(3n-5)} = \frac{3n^2 - 3n - 30}{(n-7)(3n-5)}$$

$$= \frac{3(n^2 - n - 10)}{(n-7)(3n-5)}$$

Solve each equation. Remember to check for extraneous solutions.

17)  $\frac{x^2}{(x-7)(x+2)} + \frac{1}{(x-7)(x+2)} = \frac{x+2}{x-7}$

$x^2 + 1 = x^2 + 4x + 4$   
 $-\frac{3}{4} = \frac{4x}{4}$   
 $x = \frac{-3}{4}$

18)  $6r + 24 = \frac{5r + 40}{6r} + \frac{8r^2 - 35r + 12}{6r}$

19)  $\frac{1}{(x+7)(x-3)} + 1 = \frac{x-1}{x-3}$

$1 + (x+7)(x-3) = (x+7)(x-1)$   
 $1 + x^2 + 4x - 21 = x^2 + 6x - 7$   
 $-13 = 2x$   
 $x = \frac{-13}{2}$

20)  $\frac{1}{7x-1} - \frac{3x^2 - 15x - 18}{7x^3 - x^2} = \frac{1}{7x^3 - x^2}$

21)  $\frac{n-5}{n+1} = \frac{n-8}{n+1} + \frac{1}{n^2+n}$

$n(n-5) = n(n-8) + 1$   
 $n^2 - 5n = n^2 - 8n + 1$   
 $3n = 1$   
 $n = \frac{1}{3}$

22)  $\frac{1}{a^2+3a} - \frac{a+1}{a} = \frac{a-2}{a^2+3a}$

23)  $\frac{v-2}{v} + \frac{v-3}{v-6} = \frac{1}{v}$

$(v-2)(v-6) + v(v-3) = v-6$   
 $v^2 - 8v + 12 + v^2 - 3v = v - 6$   
 $2v^2 - 12v + 18 = 0$   
 $v = 3$

24)  $\frac{1}{b+1} = \frac{b^2 - b - 12}{b^2 + b} - 1$

Simplify each expression. **★ CHALLENGE ★**

25)  $\frac{x \cdot x}{16} - \frac{4 \cdot 16}{x} = \frac{x^2 - 64}{16x} \div \frac{x^2}{16}$   
 $\frac{(x+8)(x-8)}{16x} \cdot \frac{16}{x^2}$   
 $\frac{(x+8)(x-8)}{x^3}$

26)  $\frac{3}{m+4} - \frac{3}{m+4}$

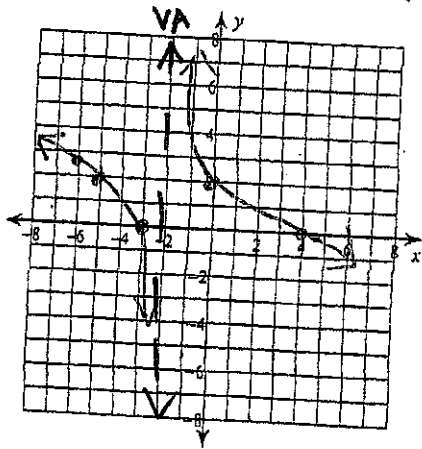
27)  $\frac{5a}{2} \div \frac{a-10}{2a}$   
 $\frac{5a}{2} \cdot \frac{2a}{a-10}$   
 $\frac{5a^2}{a-10}$

28)  $\frac{1}{3} + \frac{36}{x}$

29)  $\frac{m^2}{4 \cdot 6} - \frac{m \cdot m}{4} = \frac{m^2}{1} \div \frac{24 - m^2}{4m}$   
 $= \frac{m^2}{1} \cdot \frac{4m}{24 - m^2}$   
 $= \frac{4m^3}{24 - m^2}$

30)  $\frac{16}{u-1} - \frac{u-1}{u-1}$

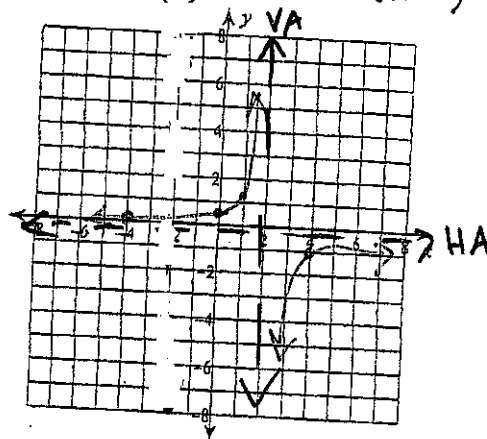
$$36) f(x) = \frac{x^2 - x - 12}{0x^2 - 3x - 6} = \frac{(x-4)(x+3)}{-3(x+2)}$$



VA:  $x = -2$     Hole: none

HA: none    Domain:  $x \in \mathbb{R}$   
 $x \neq -2$

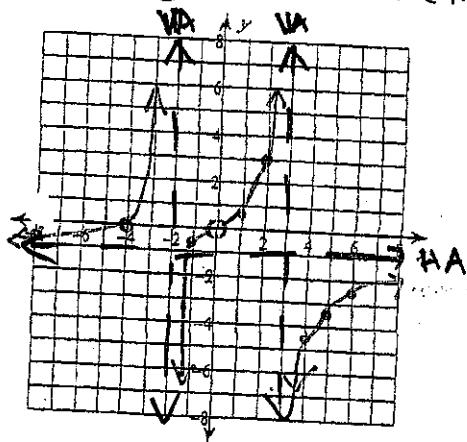
$$37) f(x) = \frac{x+4}{-4x+8} = \frac{x+4}{-4(x-2)}$$



VA:  $x = +2$     Hole:  $x = \text{none}$

HA:  $y = -\frac{1}{4}$     Domain  $x \in \mathbb{R}$   
 $x \neq +2$

$$38) f(x) = \frac{x^3 + 4x^2}{-x^3 + x^2 + 6x} = \frac{x^2(x+4)}{-x(x^2 - x - 6)} = \frac{x \cdot x(x+4)}{-x(x-3)(x+2)}$$

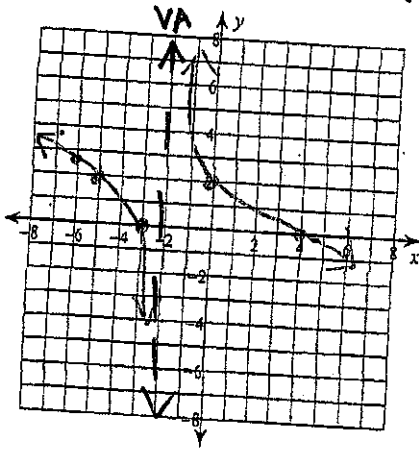


VA:  $x = 3$   
 $x = -2$     Hole:  $x = 0$

HA:  $y = -1$     Domain:  $x \in \mathbb{R}$   
 $x \neq 0, 3, -2$



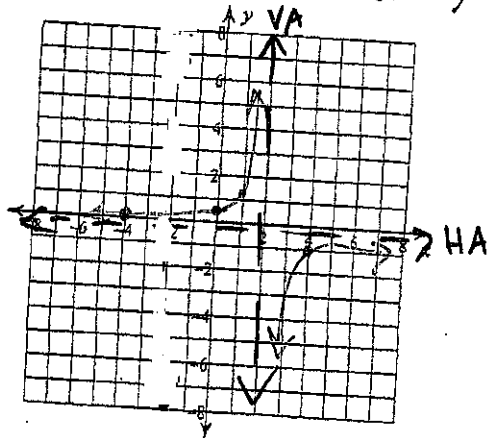
$$36) f(x) = \frac{x^2 - x - 12}{-3x - 6} = \frac{(x-4)(x+3)}{-3(x+2)}$$



VA:  $x = -2$     Hole: none

HA: none    Domain:  $x \in \mathbb{R}$   
 $x \neq -2$

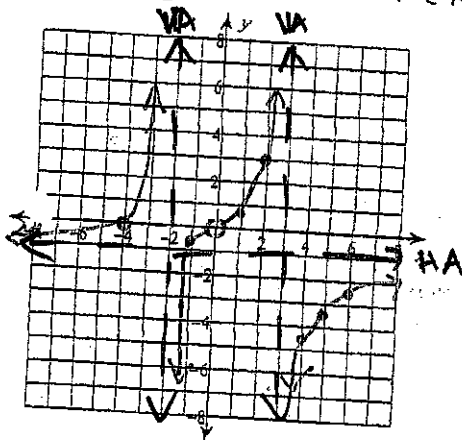
$$37) f(x) = \frac{x+4}{-4x+8} = \frac{x+4}{-4(x-2)}$$



VA:  $x = +2$     Hole:  $x = \text{none}$

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 $x \neq +2$

$$38) f(x) = \frac{x^3 + 4x^2}{-x^3 + x^2 + 6x} = \frac{x^2(x+4)}{-x(x^2 - x - 6)} = \frac{x \cdot x(x+4)}{-x(x-3)(x+2)}$$



VA:  $x = 3$   
 $x = -2$     Hole:  $x = 0$

HA:  $y = -1$     Domain:  $x \in \mathbb{R}$   
 $x \neq 0, 3, -2$